



Register Number: \_\_\_\_\_

Date: \_\_\_\_\_

**ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27**  
**BCA(DATA ANALYTICS) —IV SEMESTER**  
**SEMESTER EXAMINATION- APRIL 2022**  
**(EXAMINATION CONDUCTED IN JULY 2022)**  
**BCADA 4321: ABSTRACT ALGEBRA AND ORDINARY DIFFERENTIAL**  
**WITH HIGHER ORDER**

TIME: 2.5 Hrs

MAXIMUM MARKS: 70

This paper has 2 printed pages and 3 parts.

**Part A**

Answer all questions. More than one options may be correct.

(1 × 20 = 20)

1. What is the order of the differential equation  $y'' + 2y' = 0$   
A. 0 B. 1 C. 2 D. 3
2. What is the degree of the differential equation  $(y''')^3 + (y'')^3 + (y')^4 + y^5 = 0$   
A. 2 B. 3 C. 4 D. 5
3. Solution of the differential equation  $x dy - y dx = 0$  represents  
A. A rectangular hyperbola B. Parabola with vertex at origin C. Straight line passing through origin D. A circle whose centre is at the origin
4. Solution of the differential equations  $dy = (1 + y^2) dx$   
A.  $y = \tan(x) + c$  B.  $y = \tan(x + c)$  C.  $y = \tan^{-1}(x) + c$  D.  $\tan^{-1}(y + c) = x$
5. Solution of the differential equation  $y dy = (x + 1) dx$  is  
A.  $y^2 - x^2 + 2x - c = 0$  B.  $y^2 + x^2 - 2x - c = 0$  C.  $y^2 - x^2 - 2x - c = 0$  D. None of the above
6. The value of  $\frac{dy}{dx}$  where  $x^2 - y^2 = a^2$  is  
A.  $\frac{dy}{dx} = \frac{x}{y}$  B.  $\frac{dy}{dx} = \frac{2x}{y}$  C.  $\frac{dy}{dx} = \frac{x}{2y}$  D.  $\frac{dy}{dx} = \frac{x}{y} - a$
7. The solution for  $y'' - 3y = 0$  is  
A.  $y = ce^{3x}$  B.  $y = 3c$  C.  $y = 3y$  D. None of the above
8. Let  $\{0, 2, 4, 6, 8\}$  be a ring under addition and multiplication modulo 10. Then, a unit element is  
A. 2 B. 4 C. 6 D. 8
9.  $\mathbb{C}$  be a ring of complex numbers. Then, which of the following is a subring of  $\mathbb{C}$   
A.  $\mathbb{Z}$  B.  $n\mathbb{Z}$  C.  $\mathbb{Z}[i]$  D.  $\mathbb{Z}[x]$
10.  $\{0, 2, 4\}$  is a subring of  $\mathbb{Z}_6$  with a unit element.  
A. True B. False
11.  $\mathbb{Z}$  is isomorphic to  
A. Cyclic group of order  $n < \infty$  B. Cyclic group of infinite order C. Direct product of finite cyclic groups D. None of the above

12. The cycle (12345) can be written as  
 A. (15)(14)(13)(12) B. (12)(23)(2345) C. (123)(345)(12) D. (12)(41)(32)(31)
13. Let  $\{1, -1, i, -i\}$  be a group. The generator of this group is:  
 A. 1 B. choice -1 C.  $i$  D.  $-i$
14. Order of the permutation (1 4 2 6) is  
 A. 5 B. 8 C. 2 D. 4
15. Choose the even permutations from the following  
 A. (135) B. (12)(134)(152) C. (1356) D. (13567)
16. Choose the odd permutations from the following  
 A. (135) B. (12)(134)(152) C. (1356) D. (13567)
17. There is only one cyclic group of order 4 (upto isomorphism)  
 A. True B. False
18. How many groups of order 4 are there (upto isomorphism):  
 A. 5 B. 4 C. 3 D. 2
19. Symmetric groups are abelian.  
 A. True B. False
20. Order of  $A_6$  is  
 A.  $6!/5!$  B.  $6!/6$  C.  $6!$  D.  $6!/2$

### PART B

Answer ANY SIX questions.

(6 × 5 = 30)

21. Find  $\frac{dy}{dx}$  if  $x^3 + y^3 - 3xy^2 + 2x + 3y - 5 = 0$
22. Solve  $x \frac{dy}{dx} + y \tan(x) = \sin(2x)$  using integrating factor(IF).
23. Find the solution of  $y$  for  $y'' + y' + 5y = 0$
24. Find the solution of  $y$  for  $y''' - 6y'' + 2y' + 36y = 0$
25. Define a group homomorphism and isomorphism. Explain with two examples of each. Show that your examples are homomorphism/isomorphisms.
26. Define a ring with an example. Prove your example is a ring.
27. Define a zero-divisor. Hence Define an Integral Domain. Give an example of each.
28. Show that disjoint cycles commute.

### PART C

Answer ANY TWO questions.

(2 × 10 = 20)

29. Find the solution for the given equation:

$$y'' - 6y' + 25y = 50t^3 - 36t^2 - 64t + 18$$

30. Define a group isomorphism. Give an example of two groups that are isomorphic. State and prove Cayley's theorem.
31. Define a Ring. Hence define a field. Give an example of a ring that is not a field. Give an example of a field. For every prime  $p$ , show that  $\mathbb{Z}_p$ , the ring of integers modulo  $p$ , is a field.