Date:
Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
M.Sc MATHEMATICS - IV SEMESTER
SEMESTER EXAMINATION: APRIL 2022
(Examination conducted in July 2022)

MT0218 – Graph Theory

Time- 2 ½ hrs Max Marks-70

This question paper contains TWO printed pages

Part A

Answer any 7 questions

1. a) State and prove the fundamental theorem of Digraph theory. [5M]
   b) Prove that a digraph is strong if and only if it has a spanning closed walk. [5M]

2. a) Prove that a nontrivial connected digraph D is Eulerian if and only if $d^+(v) = d^-(v)$ for every vertex v of D. [5M]
   b) Prove that every Tournament contains a Hamiltonian path. [5M]

3. a) Prove that every non trivial connected graph has at least two vertices that are not cut vertices. [5M]
   b) Show that for any connected graph G, $\kappa(G) \leq \lambda(G) \leq \delta(G)$. [5M]

4. a) If e is a bridge in a connected graph G then show that $c(G-e) = 2$. [5M]
   b) Prove that a simple cubic 3-regular connected graph G has a cut vertex if and only if it has a cut edge. [5M]

5. a) State and prove the bounds on the sum and product of the chromatic number of a graph and its complement. [5M]
   b) Prove that a graph G with p vertices is a tree if and only if $f(G,t) = t(t-1)^{p-1}$ where $f(G,t)$ is the chromatic polynomial of G. [5M]

6. State and prove Five Colour Theorem. [10M]

7. a) Seven seniors Ben, Don, Felix, June, Kim, Lyle and Maria are looking for a job after their graduation. The Placement Officer has posted open positions for an accountant (a), consultant (c), editor (e), programmer (p), reporter (r), secretary (s) and teacher (t). Each of the seven students have applied for some of these positions:

   Ben: c, e
   Don: a, c, p, s, t
   Felix: c, r
   June: c, e, r
   Kim: a, e, p, s
   Lyle: e, r
   Maria: p, r, s, t

   Is it possible for each student to be hired for a job for which he or she has applied? Justify your answer. [5M]

b) Prove that every r-regular $(r \geq 1)$ bipartite graph has a perfect matching. [5M]

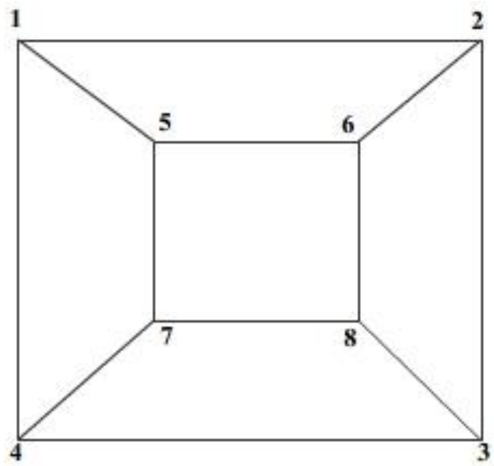
8. a) Find all 1-factors of complete graph K_6. [6M]

   b) Using Havel-Hakimi Algorithm check whether the following degree sequence D is graphical or not.

   i) $D = 5, 3, 3, 3, 3, 2, 2, 2, 1, 1, 1$

   ii) $D = 6, 6, 6, 6, 3, 3, 2, 2$ [2+2]

9. a) Define the domination number of a graph G and find the domination number of the graph given below. [5M]



   b) Define the following with example.

   (i) Independent dominating set

   (ii) Total dominating set

   (iii) Connected dominating set [2+2+1]

10. a) Define the edge independence number and edge covering number. [2 M]

    b) For every graph G containing no isolated vertices, state and prove the relation among edge independence number, edge covering number and the order of G.

    [8 M]