



Date:

Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
B.Sc. MATHEMATICS - II SEMESTER
SEMESTER EXAMINATION: APRIL 2022
(Examination conducted in July 2022)
MT218 – MATHEMATICS II

Time- 2 ½ hrs

Max Marks-70

This question paper contains **TWO** printed pages and **FOUR** parts.

I. Answer any FIVE of the following questions. [5 × 2 = 10]

1. If a is an element of a group $(G, *)$, then prove that a has a unique inverse.
2. Find the inverse of each element in (\mathbb{Z}_4, \oplus_4) using Cayley's table.
3. Define a subgroup. Find the subgroup generated by 2 in the group $(\mathbb{Z}, +)$.
4. Symbolise and negate: Some students are lazy or all students are hard working.
5. Show that $p^2 = \frac{r^3}{2a}$ is concave everywhere.
6. Find the double points of the curve $x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$.
7. Find the area bounded by the parabolas $x^2 = y$ and $y^2 = x$.
8. Solve the differential equation $\frac{dy}{dx} - y = 0$.

II. Answer any TWO of the following questions. [2 × 6 = 12]

9. Define a group. Check if $U(10)$ under multiplication modulo 10 is an abelian group or not.
10. Prove that $(G, *)$ is abelian if and only if $(a * b)^{-1} = a^{-1} * b^{-1} \quad \forall a, b \in G$.
11. Prove the following logical equivalences.
 - (i) $T[\sim p(x)] = (T[p(x)])^c$
 - (ii) $T[p(x) \wedge q(x)] = T[p(x)] \cap T[q(x)]$

III. Answer any FIVE of the following questions.

[5 × 6 = 30]

12. Find the angle between the radius vector and the tangent for the curve $r = a(1 + \cos \theta)$.
Also find the slope of the curve at $\theta = \frac{\pi}{6}$.
13. Find the pedal equation of the cardioid $r = a(1 - \cos \theta)$.
14. Find the radius of curvature at any point on the curve $xy = c^2$.
15. Find the envelope of the family of curves, $y = mx + \sqrt{a^2m^2 + b^2}$ where m is the parameter.
16. Find the entire length of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.
17. Find the area of the surface generated by revolving the curve $x = y^3$ about y axis from $y = 0$ to $y = 2$.
18. Find the volume of the solid generated by revolving an arc of the cycloid $x = a(t + \sin t)$, $y = a(1 + \cos t)$ about x axis.

IV. Answer any THREE of the following questions.

[3 × 6 = 18]

19. Solve the differential equation $(y \cos x + 2x e^y)dx + (\sin x + x^2 e^y - 1)dy = 0$.
 20. Solve the differential equation $x \frac{dy}{dx} + y = x^3 y^6$.
 21. Solve the differential equation $p^2 + (x - e^x)p - x e^x = 0$, where $p = \frac{dy}{dx}$.
 22. Find the orthogonal trajectories of the family of curves, $y^2 = 4ax$.
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