



Register Number:

Date:

**St. Joseph's College (Autonomous), Bangalore-560027**  
**B.Sc Mathematics - IV Semester**  
**Semester Examination: April 2022**  
(Examination conducted in July 2022)  
**MT 418: Mathematics IV**

**Time:** 1.5 Hours

**Max. Marks:** 35

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1. The paper contains only **ONE** printed page.
  2. Attempt any **SEVEN FULL** questions.
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- 1) Prove that the limit of the convergent sequence is unique. [5m]
- 2) If  $x_1 = \sqrt{6}$  and  $x_{n+1} = \sqrt{6x_n}$ , then show that the sequence  $\{x_n\}$  converges to 6. [5m]
- 3) Discuss the nature of the sequence  $\{x^n\}$ , where  $x$  is a real number. [5m]
- 4) If  $\sum a_n$  and  $\sum b_n$  be two series of positive terms such that
  - i)  $\sum b_n$  is convergent.
  - ii)  $a_n \leq kb_n, \forall n$ , except perhaps for the finite number of terms in the beginning, where  $k > 0$then prove that  $\sum a_n$  is also convergent. [5m]
- 5) Test the convergence of the series  $\sum_{n=1}^{\infty} (\sqrt[3]{n+1} - \sqrt[3]{n})$  [5m]
- 6) Test the convergence of the series
$$\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \dots$$
[5m]
- 7) Discuss the convergence of the series
$$\frac{1^3}{3^1} + \frac{2^3}{3^2} + \frac{3^3}{3^3} + \frac{4^3}{3^4} + \dots$$
[5m]
- 8) Discuss the convergence of the series
$$2 - \frac{3}{2} + \frac{4}{5} - \frac{5}{4} + \dots$$
[5m]
- 9) Sum the series to infinity
$$\frac{1}{6} + \frac{1.4}{6.12} + \frac{1.4.7}{6.12.18} + \dots$$
[5m]