Register Number:
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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. PHYSICS – II SEMESTER
SEMESTER EXAMINATION – APRIL 2022
(Examination conducted in July 2022)
PH8121/PH8120 : ELECTRODYNAMICS

Time: 2.5 hours **Maximum Marks:70**

This question paper contains 2 parts and 3 printed pages.

Some useful Identities:

$$\vec{\nabla}.(f\vec{A})=f(\vec{\nabla}.\vec{A})=\vec{A}.(\vec{\nabla}f)$$

$$\vec{\nabla}.\frac{\hat{r}}{r^2}=4\pi\delta^3(\vec{r})\, where \int\delta^3(r)\,d\tau=1$$

$$\vec{\nabla}.(\vec{A}\,x\,\vec{B})=\vec{B}.(\vec{\nabla}\,X\,\vec{A})-\vec{A}.(\vec{\nabla}\,X\,\vec{B})$$

Part-A

Answer any 5 questions. Each question carries 10 marks. **(10x5=50)**

1. a) A point charge 'q' is situated at a distance 'd' from the centre of a grounded conducting sphere of radius 'a' where d>a. Evaluate the potential at a point outside the sphere by finding the magnitude and position of image charge using method of images.

   b) If the position of the charge 'q' remains constant but the radius of the grounded sphere increases then what happens to the magnitude and position of image charge? (8+2)

2. a) Write down Maxwell's equations in their general form with brief significance of each equation.
   b) Now, derive these equations in their integral form. (6+4)

3. If the work done in building the current in a circuit from 0 to 'I' is given as $W=\frac{1}{2}LI^2$, then show that the work required to get this current going against back emf is $\frac{1}{2\mu_o}\int B^2 d\tau$ where $\vec{B}$ is the resulting magnetic field. This is also the energy stored in the magnetic field. Given: the flux through the source loop is $\Phi=LI$ where L is the self inductance of the loop through which current 'I' is flowing.

4. Write Maxwell's equations in a conducting medium. Show that the free charge dissipates in a characteristic time $\frac{\epsilon}{\sigma}$. Hence, from the equations with $\rho_f=0$, derive the wave equations for propagation of E.M. waves in conducting medium and interpret all the terms.

5. The potential formulation of Maxwell's equations obeys the following wave equations under

Lorentz gauge $\nabla^2 V-\mu_o\epsilon_o\frac{\partial^2 V}{\partial t^2}=\frac{-\rho}{\epsilon_o}$ and $\nabla^2 \boldsymbol{A}-\mu_o\epsilon_o\frac{\partial^2 \boldsymbol{A}}{\partial t^2}=-\mu_o \boldsymbol{J}$.

a) Show that these equations reduce to Poisson's equation and Ampere's law under static case and write the solutions of these equations i.e. equations for V,**A** under static case.

b) Now using the concept of retarded potentials, write these equations for potentials in terms of retarded time (t_r) for the non-static case. Assuming that the vector potential transforms in the same way as scalar potential, show that the scalar retarded potential (V) satisfies the inhomogeneous wave equation given above. (3+7)

6. The electric field of an arbitrarily moving charged particle is given as:

$\vec{E}(\boldsymbol{r},t)=\frac{q}{4\pi\epsilon_o}\frac{R}{(\vec{R}.\vec{u})^3}[(c^2-v^2)\vec{u}+\vec{R}x(\vec{u}\,x\,\vec{a})]$ where $\vec{u}=c\hat{R}-\vec{v}$ and

$\vec{B}(\boldsymbol{r},t)=\frac{1}{c}\hat{R}\,x\,\vec{E}(\boldsymbol{r},t)$

where **R** is the distance from retarded position **r '** to field point **r .**

a) Evaluate the expression for Poynting vector and identify the radiation energy and field energy terms.

b) Hence, evaluate the radiation energy using the expression $S_{rad}=\frac{1}{\mu_o c}E_{rad}^2\hat{R}$ where $\vec{E}_{rad}$ represents the acceleration or radiation field. Using the expressions of fields appropriately, derive an expression for the total radiated power by this point charge particle i.e the Larmor formula. (2+8)

7. a) Find the scalar product $p^{\mu}p_{\mu}$ and explain what the result signifies.
b) Explain Minkowski's Space-time diagram. (5+5)

Part-B

Answer any 4 questions. Each question carries 5 marks. (4x5=20)

8. A dielectric cube of side 'a', centred at the origin, carries a "frozen-in" polarization $\vec{P}=k\vec{r}$ where k is a constant. Find the bound surface charge, bound volume charge and the total bound charge.

9. The vector potential '**A**' of a magnetic dipole is given as $\vec{A}_{dip}(r)=(\mu_o/4\pi)(\vec{m}\times\hat{r})/r^2$ where '**m**' is magnetic dipole moment. Show that the magnetic field of this dipole can be written as: $\vec{B}(r)=(\mu_o/4\pi r^3)[3(\vec{m}.\hat{r})\hat{r}-\vec{m}]$.

10. Suppose an x-y plane forms the boundary between two linear media. An incoming monochromatic plane wave of frequency 'ω', travelling in z-direction, polarized in the plane of incidence (x-z plane) meets the boundary at an angle θ_I. It gives rise to reflected wave at angle θ_R and transmitted wave at angle θ_T where $\theta_T < \theta_I$ as velocity of wave in medium 2 v_2 is less than than in medium 1 v_1 . Assume that all the three laws of geometrical optics are obeyed. The Fresnel's equations for this polarization state are $\tilde{E}_{0R}=(\frac{\alpha-\beta}{\alpha+\beta})\tilde{E}_{0I}$ and $\tilde{E}_{0T}=(\frac{2}{\alpha+\beta})\tilde{E}_{0I}$ where $\tilde{E}_{0I},\tilde{E}_{0R},\tilde{E}_{0T}$ are the incident, reflected and transmitted amplitudes. $\alpha=\frac{\cos\theta_T}{\cos\theta_I}$ and $\beta=\frac{\mu_1 v_1}{\mu_2 v_2}$ .Assuming $\mu_1\approx\mu_2\approx\mu_0$ calculate the transmitted and reflected amplitudes and reflection and transmission

coefficients (R and T) as a function of θ_I for the air/glass interface at normal incidence, Brewster's angle, cross over angle and grazing incidence. The index of refraction of glass is 1.5 and air is 1.

11. The potentials corresponding to a charge and current distribution is given as $V(\vec{r},t)=0$ $\vec{A}(\vec{r},t)=\frac{-1}{4\pi\epsilon_o}\frac{qt}{r^2}\hat{r}$ where symbols have their usual meaning. If this potential under a valid gauge transformation changes to $\vec{A}'=0$ and $V'=\frac{1}{4\pi\epsilon_o}\frac{q}{r}$ then find the value of λ for this to be a valid gauge transformation.

12. An atomic clock is placed in a jet plane. The clock measures a time interval of 3600 s when the jet moves with speed 400 m/s. How much larger a time interval does an identical clock held by an observer at rest on the ground measure?

13. A 125-turn rectangular coil of wire with sides of 25 and 40 cm rotates about a horizontal axis in a vertical magnetic field of magnitude 3.5 mT. How fast must this coil rotate for the induced emf to reach 5 volts?