| Test Paper | : Il |
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| Test Subject | $:$ MATHEMATICAL SCIENCE |
| Test Subject Code $:$ K-2616 |  |

Test Booklet Serial No. :
OMR Sheet No. :
Roll No.


## Name \& Signature of Invigilator/s

Signature :

## Name

# Paper : II <br> Subject : MATHEMATICAL SCIENCE 

Time: 1 Hour 15 Minutes
Maximum Marks : 100
Number of Pages in this Booklet : $\mathbf{8} \quad$ Number of Questions in this Booklet : $\mathbf{5 0}$






 స్డిలళరైబొఱ.









లుదాळరణ: (A) B
(C) సరియోద లుత్తరవాగిద్దిగ.


 అడర మోల్యమూజ్ట మృజలాగుబ్రిల్ల్











 లుజయినలగజస్ను నిజొధిసలాగిది.
13. 戸ం ఆల్లద లుత్తరగళిగి ముణ అంః ఇరువుదల్ల.
 చండుబందల్లి, ఇంగ్లిల్షో ఆపృత్తిగళల్లిపుపుదిల ఆంతిముఐిందు జరిగణిసబెలచు.

## Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of fifty multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
(i) To have access to the Question Booklet, tear off the paper seal on the edge of the cover page. Do not accept a booklet without sticker seal or open booklet.
(ii) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
Example: (A) B (D)
where $(C)$ is the correct response.
5. Your responses to the questionsare to be indicated in the OMR Sheet kept inside the Paper I Booklet only. If you mark at any place other than in the circles in the OMR Sheet, it will not be evaluated.
6. Read the instructions given in OMR carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away question booklet and carbon copy of OMR Answer Sheet after the examination.
11. Use only Blue/Black Ball point pen.
12. Use of any calculator, Electronic gadgets or log table etc., is prohibited.
13. There is no negative marks for incorrect answers.
14. In case of any discrepancy found in the Kannada translation of a question booklet the question in English version shall be taken as final.

## MATHEMATICS <br> Paper - II

Note : This paper contains fifty (50) objective type questions. Each question carries two (2) marks. All questions are compulsory.

1. Let $a_{n}=(-1)^{n} \frac{\log \left(n^{4}+1\right)}{n^{2}+1}$. Then the sequence $\left\{a_{n}\right\}$
(A) is not convergent
(B) converges to 0
(C) converges to a non-zero limit
(D) oscillates between $-\infty$ and $\infty$
2. If $f, g$ are real-valued functions, then $\max .(\mathrm{f}, \mathrm{g})=$
(A) $\frac{f+g-|f-g|}{2}$
(B) $\frac{|f-g|-|f+g|}{2}$
(C) $\frac{f+g+|f-g|}{2}$
(D) $\frac{|\mathrm{f}-\mathrm{g}|+|\mathrm{f}-\mathrm{g}|}{2}$
3. Let $\mathrm{f}:[\mathrm{a}, \mathrm{b}] \rightarrow \mathbb{R}$ take the value 1 at rational points and -1 at irrational points. Then the upper and the lower Riemann integrals are respectively
(A) $b-a$ and $-(b-a)$
(B) b - a and 0
(C) 1 and -1
(D) 1 and 0
4. Which one of the following has positive Lebesgue measure in $\mathbb{R}$ ?
(A) The cantor set
(B) $Z$
(C) Q
(D) $(\mathrm{a}, \mathrm{b})-\mathrm{Q}$, where $\mathrm{a}<\mathrm{b}$
5. The closed unit ball of a normed linear space $X$ is norm compact if and only if
(A) X is finite dimensional
(B) $X$ is reflexive
(C) $X$ is a Hilbert space
(D) X is a Banach space
6. Suppose the matrix

$$
A=\left[\begin{array}{ccc}
3 & x+2 i & y i \\
3-2 i & 0 & 1+z i \\
y i & 1-x i & -1
\end{array}\right] \text { is Hermitian. }
$$

Then ( $x, y, z$ ) must be
(A) $(0,3,3)$
(B) $(3,0,3)$
(C) $(3,3,0)$
(D) $(0,0,3)$
7. The 2-dimensional subspaces of $\mathbb{R}^{3}$ can be geometrically described as
(A) All planes
(B) All planes passing through the origin
(C) All lines passing through the origin
(D) The only planes $x=0, y=0, z=0$
8. Let $A=\left[\begin{array}{lll}1 & 1 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 3\end{array}\right]$. The characteristic polynomial of $A$ is given by
(A) $\pm(\lambda+1)(\lambda-2)(\lambda-3)$
(B) $\pm(\lambda-1)(\lambda+2)(\lambda-3)$
(C) $\pm(\lambda-1)(\lambda-2)(\lambda+3)$
(D) $\pm(\lambda-1)(\lambda-2)(\lambda-3)$
9. Let M be the set of all $2 \times 2$ matrices with real entries and let $f: M \rightarrow \mathbb{R}$ be the determinant map

Then
(A) $f$ is one-one and onto
(B) $f$ is neither one-one nor onto
(C) $f$ is one-one but not onto
(D) $f$ is onto but not one-one
10. The dimension of the vector space $M=\left\{\left[a_{i j}\right]_{m \times n}: a_{i j} \in \mathbb{C}\right\}$ over the field $\mathbb{R}$ of real numbers is
(A) $m+n$
(B) $2 m n$
(C) $2(m+n)$
(D) mn
11. The radius of convergence of the power series $\sum 2^{n} z^{n}$ is
(A) 1
(B) 0
(C) 2
(D) $1 / 2$
12. Let $f$ be an analytic function in $\mathbb{C}$. $A$ sufficient condition for $f$ to be constant is
(A) $f\left(\frac{1}{n}\right)=0$ for $n=1,2,3 \ldots$.
(B) $f(n)=0$ for $n=0, \pm 1, \pm 2, \ldots \ldots$
(C) $f(0)=0$
(D) $f(z)=0$ for $z=0, \pm 1, \pm i$
13. If $1, \omega, \omega^{2}$ are the cube roots of unity, then the value of $\left(\omega+\omega^{2}\right)^{3}$ is
(A) 0
(B) 1
(C) -1
(D) 3
14. The modulus of the complex number $e^{z}$ for any complex number $z$ is
(A) $e^{|z|}$
(B) $e^{R e z}$
(C) $e^{\operatorname{lm} z}$
(D) $\mathrm{e} \mid z$
15. The locus of $|z+1|=|z-1|$ in the complex plane is
(A) A straight line
(B) A circle
(C) An ellipse
(D) A parabola
16. If $x$ is a positive integer satisfying $x \equiv 3 \bmod 7$ and $x \equiv-3 \bmod 11$, then
(A) $x=3$
(B) No such $x$ exists
(C) Exactly one such $x$ exists
(D) There are infinitely many such $x$
17. The number of primitive $11^{\text {th }}$ roots of unity is
(A) 1
(B) 2
(C) 10
(D) 5
18. The number of subgroups of a cyclic group of order 20 is
(A) 6
(B) 2
(C) 10
(D) 1
19. For which prime $P$ in the following, both - 1 and 2 are quadratic residues modulo P ?
(A) 7
(B) 17
(C) 11
(D) 13
20. Let $P$ be prime and let $F$ be a field with $P^{2}$ number of elements. Then the number of ideals in $F$ is
(A) 1
(B) $P^{2}$
(C) 2
(D) $P^{2}-1$
21. In $X=[0,1]$ with the topology given by the metric $d(x, y)=|x-y|,[0,1 / 2)$ is
(A) Closed
(B) Open
(C) Both open and closed
(D) Neither open nor closed
22. In $X=[0,2] \cup\{3\}$ with the topology given by $\mathrm{d}(\mathrm{x}, \mathrm{y})=|\mathrm{x}-\mathrm{y}|$,
Int. $([1,2] \cup\{3\})=$
(A) $(1,2)$
(B) $(1,2) \cup\{3\}$
(C) $(1,2] \cup\{3\}$
(D) $[1,2] \cup\{3\}$
23. The only compact subsets of the real line $\mathbb{R}$ with the usual distance metric are
(A) $\phi$ and $\mathbb{R}$
(B) Closed intervals
(C) Finite union of closed intervals
(D) Closed and bounded sets
24. The number of linearly independent solutions of the form $y=x^{r}$ of the differential equation $x^{3} \frac{d^{3} y}{d x^{3}}-6 x \frac{d y}{d x}+12 y=0$ is
(A) 0
(B) 1
(C) 2
(D) 3
25. The partial differential equation
$x^{2} \frac{\partial^{2} z}{\partial x^{2}}-\left(y^{2}-1\right) x \frac{\partial^{2} z}{\partial x \partial y}+y(y-1)^{2} \frac{\partial^{2} z}{\partial y^{2}}+$ $x \frac{\partial z}{\partial x}+y \frac{\partial z}{\partial y}=0$ is hyperbolic in a region in the $x y$ - plane if
(A) $x \neq 0$ and $y=1$
(B) $x=0$ and $y \neq 1$
(C) $x \neq 0, y \neq 1$ and $y \neq 2$
(D) $x=0$ and $y=1$
26. The partial differential equation of the set of all right circular cones whose axes coincide with $z$-axis is
(A) $x \frac{\partial z}{\partial x}=y \frac{\partial z}{\partial y}$
(B) $y \frac{\partial z}{\partial x}=x \frac{\partial z}{\partial y}$
(C) $x^{2} \frac{\partial^{2} z}{\partial x^{2}}=y^{2} \frac{\partial^{2} z}{\partial y^{2}}$
(D) $y^{2} \frac{\partial^{2} z}{\partial x^{2}}=x^{2} \frac{\partial^{2} z}{\partial y^{2}}$
27. In the motion of a two particle system, if two particles are connected by a rigid weightless rod of constant length, then the number of degrees of freedom of the system is
(A) 5
(B) 2
(C) 3
(D) 6
28. Extremal for the variational problem $I[y(x)]=\int_{x_{0}}^{x_{1}} \frac{d y}{d x}\left(1+x^{2} \frac{d y}{d x}\right) d x$ is a solution of the differential equation
(A) $x^{2} \frac{d y}{d x}+\frac{d y}{d x}=0$
(B) $\frac{d^{2} y}{d x^{2}}+2 \frac{d y}{d x}=0$
(C) $x \frac{d^{2} y}{d x^{2}}+2 \frac{d y}{d x}=0$
(D) $\frac{d^{2} y}{d x^{2}}+2 x \frac{d y}{d x}=0$
29. In the solution of the following system of linear equations by Gauss elimination use partial pivoting $5 x+y+2 z=34$; $4 y-3 z=12 ; 10 x-2 y+z=-4$. The pivots for elimination of $x$ and $y$ are
(A) 10, 4
(B) 10,2
(C) 5,4
(D) $5,-4$
30. Which of the following matrix admits a Cholesky decomposition?
(A) $\left[\begin{array}{ll}1 & i \\ i & 1\end{array}\right]$
(B) $\left[\begin{array}{cc}1 & 2 i \\ -2 i & 1\end{array}\right]$
(C) $\left[\begin{array}{rc}1 & -2 \\ -2 & 5\end{array}\right]$
(D) $\left[\begin{array}{ll}1 & 1 \\ 2 & 2\end{array}\right]$
31. For a negatively skewed distribution, the relationship among mean, median and mode is
(A) Mode $>$ Mean $>$ Median
(B) Mean $>$ Median $>$ Mode
(C) Mode > Median > Mean
(D) Mode $=$ Mean $=$ Median
32. Let $\Omega$ be a sample space, $A$ and $B$ are events such that $P(A)=1, P(B)=0$. Then one of the following is necessarily TRUE
(A) $\mathrm{P}(\mathrm{A} \cup \mathrm{B})<1$ and $\mathrm{P}(\mathrm{A} \cap \mathrm{B})=0$
(B) $\mathrm{P}(\mathrm{A} \cup \mathrm{B})=1$ and $\mathrm{P}(\mathrm{A} \cap \mathrm{B})=0$
(C) $A=\Omega$ and $B=\phi$
(D) $B^{C}=A$
33. A random variable $X$ with mean 5 and variance 4 , then mean and standard deviation of $4 x+5$ are
(A) 21,16
(B) 25, 64
(C) 25,16
(D) 25,8
34. Let $X$ be a two point distribution taking values 0 and 2 with equal probability. Then its characteristic function is
(A) $\frac{1}{2}\left(1+\mathrm{e}^{2 \mathrm{t}}\right)$
(B) $\frac{1}{2}+\mathrm{e}^{\mathrm{i} 2 \mathrm{t}}$
(C) $\frac{1}{2}\left(1+e^{i 2 t}\right)$
(D) $1+\frac{1}{2} e^{i 2 t}$
35. $\phi(\mathrm{t})$ is a characteristic function of a random variable. Then one of the following NEED NOT be satisfied
(A) $|\phi(t)| \leq 1$
(B) $\phi(\mathrm{t})$ is continuous
(C) $\phi(\mathrm{t})$ always exists
(D) $\phi(\mathrm{t})=\phi(-\mathrm{t})$
36. A necessary condition for WLLN for a sequence of iid random variable $\left\{X_{n}\right\}$ is
(A) $E X_{1}$ exists and finite
(B) $n P\left[\left|X_{1}\right|>n\right] \rightarrow 0$ as $n \rightarrow \infty$
(C) $V\left(X_{1}\right)$ is finite
(D) $X_{1}$ is symmetric about 0
37. Let $\left\{X_{n}\right\}$ be a Markov chain with state space $\{1,2,3\}$ and transition probability matrix
$P=\left[\begin{array}{ccc}1 / 3 & 1 / 3 & 1 / 3 \\ 1 / 2 & 0 & 1 / 2 \\ 0 & 0 & 1\end{array}\right]$.
State 3 is
(A) Absorbing and transient
(B) Absorbing and persistent
(C) Communicating with all the states
(D) Has period 2
38. Chapman-Kolmogorov equation in a Markov-chain is used to compute
(A) Communicating states
(B) Absorbing states
(C) Higher order transition probabilities
(D) One state transition probability
39. $\mathrm{N}_{1}(\mathrm{t})$ and $\mathrm{N}_{2}(\mathrm{t})$ are independent Poisson processes, then for any fixed $t$,
$\mathrm{N}_{1}(\mathrm{t}) \mid\left[\mathrm{N}_{1}(\mathrm{t})+\mathrm{N}_{2}(\mathrm{t})=\mathrm{n}\right]$ follows
(A) Poisson distribution
(B) Gaussian distribution
(C) Negative binomial distribution
(D) Binomial distribution
40. Queuing process is a
(A) Poisson process
(B) Pure birth process
(C) Erlang process
(D) Birth-death process
41. Based on two independent random samples from continuous distributions $F_{x}(x)$ and $G_{y}(x)$ with sample sizes 4 and 5 respectively, the value of the Wilcoxon rank-sum statistic, that is, sum of ranks of Y observations is 20 . Then the value of Mann-Whitney statistic is
(A) 6
(B) 7
(C) 5
(D) 8
42. The consistency of the test statistic for testing $H_{0}: \theta \in\left(H_{0}\right.$ against $H_{1}: \theta \in\left({ }_{( }\right)_{1}$ implies that
(A) Size of that test goes to zero as $n \rightarrow \infty$
(B) Power of the test is larger than the size of the test
(C) Power of the test statistic approaches 1 as $n \rightarrow \infty$
(D) Size of the test is less than power of the test
43. Let $\left\{X_{1}, X_{2}, \ldots . ., X_{n}\right\}$ be a random sample from a continuous distribution function $F$ and let $F_{n}$ be the corresponding empirical distribution function.

Let
$D_{n}^{+}=\sup _{x}\left[F_{n}(x)-F(x)\right]$
$D_{n}^{-}=\sup _{x}\left[F(x)-F_{n}(x)\right]$
$D_{n}=\operatorname{Max} .\left(D_{n}^{+}, D_{n}^{-}\right)$
Which of the following is TRUE ?
(A) $D_{n}^{+}, D_{n}^{-}$and $D_{n}$ are distribution-free over the class of all discrete distributions
(B) $D_{n}^{+}, D_{n}^{-}$and $D_{n}$ distribution-free over all continuous distributions
(C) $D_{n}$ is distribution-free but not $\mathrm{D}_{\mathrm{n}}^{+}$and $\mathrm{D}_{\mathrm{n}}^{-}$
(D) $\mathrm{D}_{\mathrm{n}}^{+}$and $\mathrm{D}_{\mathrm{n}}^{-}$are distribution-free but not $D_{n}$
44. Let $\left\{X_{i}\right\}_{i=1}^{n}$ be i.i.d. $N\left(\theta_{1}, \theta_{2}\right)$ r.v.'s, then, the consistent estimator of $\theta_{1}^{2}+\theta_{2}$ is
(A) $\frac{1}{n} \sum_{i=1}^{n} X_{i}^{2}$
(B) $\bar{x}^{2}$
(C) $\frac{1}{n} \sum_{i=1}^{n}\left|X_{i}-\bar{X}\right|^{2}$
(D) $\left(\frac{1}{n-1} \sum x_{i}\right)^{2}$
45. Canonical Correlation Analysis is used to
(A) Quantify the association between two sets of multivariate data
(B) Reduce the dimension of exogenous variables
(C) Extract best linear combination from one set of data
(D) Identify the prominent components in one set of variables
46. In a $2^{3}$ factorial experiment with three factors $A, B, C$ at two levels $a_{0}, b_{0}, c_{0}$ and $a, b, c$. Then the contrast for the interaction $A B$ can be obtained from the product
(A) $\left(a_{1}-a_{0}\right)\left(b_{1}-b_{0}\right)\left(c_{1}-c_{0}\right)$
(B) $\left(a_{1}-a_{0}\right)\left(b_{1}-b_{0}\right)\left(c_{1}+c_{0}\right)$
(C) $\left(a_{1}+a_{0}\right)\left(b_{1}+b_{0}\right)\left(c_{1}-c_{0}\right)$
(D) $\left(a_{1}-a_{0}\right)\left(b_{1}+b_{0}\right)\left(c_{1}+c_{0}\right)$
47. The null hypothesis in the ANOVA of a linear regression model is
(A) Means of explanatory variables are equal
(B) Response variable is linearly related to the regressors
(C) Response variable is not depending on any of the regressors
(D) Variance of the response variable is a constant
48. The best linear unbiased predictor of mean response in a regression model for a specified new observation on the regressor is obtained by
(A) Replacing the model parameters with their unbiased estimators
(B) Replacing the model parameters with their best linear unbiased estimators
(C) Replacing the regression coefficients with generalised least squares estimators
(D) Replacing the regression coefficients with their consistent estimators
49. In SRSWOR of size $n$, the bias in the ratio estimator $\hat{R}$ is
(A) Less than or equal to $\frac{\operatorname{SE}(\hat{R}) \operatorname{SE}(\bar{x})}{\bar{X}}$
(B) Greater than or equal to $\frac{S E(\hat{R})}{\bar{X}}$
(C) Less than or equal to $\frac{\operatorname{SE}(\bar{x})}{\bar{X}}$
(D) Greater than $\frac{\operatorname{SE}(\hat{R}) \cdot \operatorname{SE}(\bar{x})}{\bar{X}}$
50. Stratified sampling is more efficient than simple random sampling because
(A) Population is divided into strata in such a way that the units within the strata are homogeneous and units between strata are heterogenous, leading to minimal internal variation
(B) Population is subdivided into mutually exclusive and exhaustive subpopulations
(C) Strata are small and hence sampling variation is small
(D) Samples are selected from few strata

[^0]
[^0]:     Space for Rough Work

