Test Paper : **III**

Test Subject : **MATHEMATICAL SCIENCE**

Test Subject Code : **K-2615**

Test Booklet Serial No. : ____________________

OMR Sheet No. : ____________________

Roll No. | | | | | | | | |

(Figures as per admission card)

Name & Signature of Invigilator/s

Signature : ____________________

Name : ____________________

Paper : III

Subject : MATHEMATICAL SCIENCE

Time : 2 Hours 30 Minutes — Maximum Marks : 150

Number of Pages in this Booklet : **16** — Number of Questions in this Booklet : **75**

ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಸೂಚನೆಗಳು

1. ಈ ಪುಟದ ಮೇಲ್ತುದಿಯಲ್ಲಿ ಒದಗಿಸಿದ ಸ್ಥಳದಲ್ಲಿ ನಿಮ್ಮ ರೋಲ್ ನಂಬರನ್ನು ಬರೆಯಿರಿ.
2. ಈ ಪತ್ರಿಕೆಯು ಬಹು ಆಯ್ಕೆ ವಿಧದ ಎಪ್ಪತ್ತೈದು ಪ್ರಶ್ನೆಗಳನ್ನು ಒಳಗೊಂಡಿದೆ.
3. ಪರೀಕ್ಷೆಯ ಪ್ರಾರಂಭದಲ್ಲಿ, ಪ್ರಶ್ನೆ ಪುಸ್ತಿಕೆಯನ್ನು ನಿಮಗೆ ನೀಡಲಾಗುವುದು. ಮೊದಲ 5 ನಿಮಿಷಗಳಲ್ಲಿ ನೀವು ಪುಸ್ತಿಕೆಯನ್ನು ತೆರೆಯಲು ಮತ್ತು ಕೆಳಗಿನಂತೆ ಕಡ್ಡಾಯವಾಗಿ ಪರೀಕ್ಷಿಸಲು ಕೋರಲಾಗಿದೆ.
   (i) ಪ್ರಶ್ನೆ ಪುಸ್ತಿಕೆಗೆ ಪ್ರವೇಶಾವಕಾಶ ಪಡೆಯಲು, ಈ ಹೊದಿಕೆ ಪುಟದ ಅಂಚಿನ ಮೇಲಿರುವ ಪೇಪರ್ ಸೀಲನ್ನು ಹರಿಯಿರಿ. ಸ್ಟಿಕ್ಕರ್ ಸೀಲ್ ಇಲ್ಲದ ಅಥವಾ ತೆರೆದ ಪುಸ್ತಿಕೆಯನ್ನು ಸ್ವೀಕರಿಸಬೇಡಿ.
   (ii) **ಪುಸ್ತಿಕೆಯಲ್ಲಿನ ಪ್ರಶ್ನೆಗಳ ಸಂಖ್ಯೆ ಮತ್ತು ಪುಟಗಳ ಸಂಖ್ಯೆಯನ್ನು ಮುಖಪುಟದ ಮೇಲೆ ಮುದ್ರಿಸಿದ ಮಾಹಿತಿಯೊಂದಿಗೆ ತಾಳೆ ನೋಡಿರಿ. ಪುಟಗಳು/ಪ್ರಶ್ನೆಗಳು ಕಾಣೆಯಾದ, ಅಥವಾ ದ್ವಿಪ್ರತಿ ಅಥವಾ ಅನುಕ್ರಮವಾಗಿಲ್ಲದ ಅಥವಾ ಇತರ ಯಾವುದೇ ವ್ಯತ್ಯಾಸದ ದೋಷಪೂರಿತ ಪುಸ್ತಿಕೆಯನ್ನು ಕೂಡಲೆ 5 ನಿಮಿಷದ ಅವಧಿ ಒಳಗೆ, ಸಂವೀಕ್ಷಕರಿಂದ ಸರಿ ಇರುವ ಪುಸ್ತಿಕೆಗೆ ಬದಲಾಯಿಸಿಕೊಳ್ಳಬೇಕು. ಆ ಬಳಿಕ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ಬದಲಾಯಿಸಲಾಗುವುದಿಲ್ಲ, ಯಾವುದೇ ಹೆಚ್ಚು ಸಮಯವನ್ನೂ ಕೊಡಲಾಗುವುದಿಲ್ಲ.**
4. ಪ್ರತಿಯೊಂದು ಪ್ರಶ್ನೆಗೂ (A), (B), (C) ಮತ್ತು (D) ಎಂದು ಗುರುತಿಸಿದ ನಾಲ್ಕು ಪರ್ಯಾಯ ಉತ್ತರಗಳಿವೆ. ನೀವು ಪ್ರಶ್ನೆಯ ಎದುರು ಸರಿಯಾದ ಉತ್ತರದ ಮೇಲೆ, ಕೆಳಗೆ ಕಾಣಿಸಿದಂತೆ ಅಂಡಾಕೃತಿಯನ್ನು ಕಪ್ಪಾಗಿಸಬೇಕು.
   ಉದಾಹರಣೆ : (A) (B) ● (D)
   (C) ಸರಿಯಾದ ಉತ್ತರವಾಗಿದ್ದಾಗ.
5. ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಗಳನ್ನು, **ಪತ್ರಿಕೆ III ಪುಸ್ತಿಕೆಯೊಳಗೆ ಕೊಟ್ಟಿರುವ OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಮಾತ್ರವೇ ಸೂಚಿಸತಕ್ಕದ್ದು.** OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿನ ಅಂಡಾಕೃತಿ ಹೊರತುಪಡಿಸಿ ಬೇರೆ ಯಾವುದೇ ಸ್ಥಳದಲ್ಲಿ ಗುರುತಿಸಿದರೆ, ಅದರ ಮೌಲ್ಯಮಾಪನ ಮಾಡಲಾಗುವುದಿಲ್ಲ.
6. OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಕೊಟ್ಟ ಸೂಚನೆಗಳನ್ನು ಜಾಗರೂಕತೆಯಿಂದ ಓದಿರಿ.
7. ಎಲ್ಲಾ ಕರಡು ಕೆಲಸವನ್ನು ಪುಸ್ತಿಕೆಯ ಕೊನೆಯಲ್ಲಿ ಮಾಡತಕ್ಕದ್ದು.
8. ನಿಮ್ಮ ಗುರುತನ್ನು ಬಹಿರಂಗಪಡಿಸಬಹುದಾದ ನಿಮ್ಮ ಹೆಸರು ಅಥವಾ ಯಾವುದೇ ಚಿಹ್ನೆಯನ್ನು, ಸಂಗತವಾದ ಸ್ಥಳ ಹೊರತು ಪಡಿಸಿ, OMR ಉತ್ತರ ಹಾಳೆಯ ಯಾವುದೇ ಭಾಗದಲ್ಲಿ ಬರೆದರೆ, ನೀವು ಅನರ್ಹತೆಗೆ ಬಾಧ್ಯರಾಗಿರುತ್ತೀರಿ.
9. ಪರೀಕ್ಷೆಯು ಮುಗಿದನಂತರ, ಕಡ್ಡಾಯವಾಗಿ OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ಸಂವೀಕ್ಷಕರಿಗೆ ನೀವು ಹಿಂತಿರುಗಿಸಬೇಕು ಮತ್ತು ಪರೀಕ್ಷಾ ಕೊಠಡಿಯ ಹೊರಗೆ OMR ನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ಕೊಂಡೊಯ್ಯಕೂಡದು.
10. ಪರೀಕ್ಷೆಯ ನಂತರ, ಪರೀಕ್ಷಾ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ಮತ್ತು ನಕಲು OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ತೆಗೆದುಕೊಂಡು ಹೋಗಬಹುದು.
11. **ನೀಲಿ/ಕಪ್ಪು ಬಾಲ್‌ಪಾಯಿಂಟ್ ಪೆನ್ ಮಾತ್ರವೇ ಉಪಯೋಗಿಸಿರಿ.**
12. **ಕ್ಯಾಲ್ಕುಲೇಟರ್ ಅಥವಾ ಲಾಗ್ ಟೇಬಲ್ ಇತ್ಯಾದಿಯ ಉಪಯೋಗವನ್ನು ನಿಷೇಧಿಸಲಾಗಿದೆ.**
13. **ಸರಿ ಅಲ್ಲದ ಉತ್ತರಗಳಿಗೆ ಋಣ ಅಂಕ ಇರುವುದಿಲ್ಲ.**
14. **ಕನ್ನಡ ಮತ್ತು ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಗಳ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಗಳಲ್ಲಿ ಯಾವುದೇ ರೀತಿಯ ವ್ಯತ್ಯಾಸಗಳು ಕಂಡುಬಂದಲ್ಲಿ, ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಗಳಲ್ಲಿರುವುದೇ ಅಂತಿಮವೆಂದು ಪರಿಗಣಿಸಬೇಕು.**

Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of seventy five multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of the cover page. Do not accept a booklet without sticker seal or open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the oval as indicated below on the correct response against each item.
   Example : (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the question of Paper III are to be indicated in the **OMR Sheet kept inside the Booklet**. If you mark at any place other than in the ovals in OMR Answer Sheet, it will not be evaluated.
6. Read the instructions given in OMR carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away question booklet and carbon copy of OMR Answer Sheet soon after the examination.
11. **Use only Blue/Black Ball point pen.**
12. **Use of any calculator or log table etc., is prohibited.**
13. **There is no negative marks for incorrect answers.**
14. **In case of any discrepancy found in the Kannada translation of a question booklet the question in English version shall be taken as final.**

MATHEMATICAL SCIENCE

PAPER – III

Note : This paper contains **seventy-five (75)** objective type questions. **Each** question carries **two (2)** marks. **All** questions are **compulsory**.

1. $\lim_{n\to\infty} \frac{n}{(n!)^{1/n}} =$

(A) 1 (B) 0
(C) $+\infty$ (D) e

2. The function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & , x = 0 \end{cases}$$ is

(A) continuous everywhere
(B) continuous only at $x = 0$
(C) not continuous at $x = 0$
(D) differentiable at $x = 0$

3. Which one of the following series is convergent ?

(A) $\sum_{n=1}^{\infty} \left(\frac{2n+5}{3n+1}\right)$

(B) $\sum_{n=1}^{\infty} \left(\frac{1.3.5...(2n-1)}{2.4.6...(2n)}\right)^2$

(C) $\sum_{n=1}^{\infty} \frac{1}{n^{n+\frac{1}{n}}}$

(D) $\sum_{n=2}^{\infty} \frac{1}{(\log n)^{\log n}}$

4. The sum of the series

$$1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \ldots$$ is

(A) e^2

(B) 3

(C) $\sqrt{5}$

(D) $2\sqrt{2}$

5. $\int_0^1 \left(\log\frac{1}{x}\right)^{-1/2} dx =$

(A) $\frac{\sqrt{\pi}}{4}$

(B) $\Gamma\left(3/2\right)$

(C) $\Gamma\left(1/2\right)$

(D) π

6. Let $f(x, y) = y^2 + 4xy + 3x^2 + x^3$. Then

(A) (0, 0) is a saddle point of f
(B) (0, 0) is a minimum point of f
(C) (0, 0) is a maximum point of f
(D) $\left(2/3, -4/3\right)$ is a maximum point of f

7. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$. Then which one of the following statements is true ?

(A) If f is continuous at a point, then f has partial derivatives at that point

(B) If f has partial derivatives at a point, then f is continuous at that point

(C) If f has partial derivatives at a point, then f is differentiable at that point

(D) If f is differentiable at a point, then f has partial derivatives at that point

8. If $u = \frac{yz}{x}$, $v = \frac{xz}{y}$, $w = \frac{xy}{z}$, then the Jacobian $J\left(\frac{u, v, w}{x, y, z}\right) =$

(A) 0

(B) 2

(C) 4

(D) 6

9. Let X be an infinite set. For $x, y \in X$, define

$$d(x, y) = \begin{cases} 1, & \text{if } x \neq y \\ 0, & \text{if } x = y \end{cases}$$

Then which subsets of X are compact in the metric space (X, d) ?

(A) All subsets of X

(B) All finite subsets of X

(C) Only empty set

(D) Only the set X

10. The infimum, liminf, limsup and supremum of the sequence

$$x_n = \begin{cases} 2 - \frac{1}{n}, & \text{if n is odd} \\ 3 + \frac{1}{n^2}, & \text{if n is even} \end{cases}$$

are respectively

(A) 1, 1, 3, 3

(B) 2, 2, 3, 3

(C) $\frac{13}{4}$, 3, 2, 1

(D) 1, 2, 3, $\frac{13}{4}$

11. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear mapping defined by T (a, b, c) = (0, a, b). Then $\text{Ker}T^2$ is equal to

(A) $\{(x, y, z) \mid z = y = 0\}$

(B) $\{(x, y, z) \mid x = y = z\}$

(C) $\{(x, y, z) \mid x = 0\}$

(D) $\{(x, y, z) \mid x \neq 0\}$

12. Let V be the vector space of all polynomials with real coefficients of degree ≤ 2. Let $f(x) = x - 1$, $g(x) = x + 1$, $h(x) = x^2 - 1$ and $j(x) = x^2 + 1$. Then $\{f(x), g(x), h(x), j(x)\}$ is

(A) linearly independent

(B) linearly dependent because $f(x)\, g(x) = h(x)$

(C) linearly dependent because $f(x) - g(x) + j(x) = h(x)$

(D) linearly dependent because $f(x)\, g(x) + 2 = j(x)$

13. If U and V are two vector spaces over the same field F, U is finite dimensional and if T : U $\rightarrow$ V is a linear transformation, then Rank (T) + Nullity (T) =

(A) dim V

(B) dim U

(C) dim V + dim U

(D) dim V. dim U

14. In the ring of Gaussian integers $\mathbb{Z}$ [i], which one of the following statements is not true ?

(A) $\pm 1, \pm i$ are the only units

(B) 1 + i is irreducible

(C) 2 + 3i is irreducible

(D) 17 is irreducible

15. The roots of the equation

$x^{18} + 4x^{14} + 3x + 10 \equiv 0 \pmod{21}$ are

(A) 3, 12, 14, 18

(B) 2, 4, 8, 10

(C) 5, 10, 17, 19

(D) 3, 5, 7, 11

16. Let a and b be two positive real numbers. Then the number of real eigen values of the matrix $\begin{pmatrix} a & 1 \\ 2 & b \end{pmatrix}$ is

(A) 0

(B) 1

(C) 2

(D) a + b

17. Which one of the following functions is not analytic ?

(A) $f(z) = \sin z$

(B) $f(z) = 1 + z + z^{10}$

(C) $f(z) = e^{z}$

(D) $f(z) = \bar{z}$

18. For the function $f(z) = \dfrac{z - \sin z}{z^3}$, the point z = 0 is

(A) a pole of order 3

(B) a pole of order 2

(C) an essential singularity

(D) a removable singularity

19. The residue of the function

$f(z) = \frac{z^3}{z^2 - 1}$ at $z = \infty$ is

(A) 0

(B) 1

(C) i

(D) – 1

20. If $i = \sqrt{-1}$, then $i^i =$

(A) $\{e^{\frac{\pi}{2}+2k\pi} | k \in \mathbb{Z} \}$

(B) $\{e^{-\left(\frac{\pi}{3}+2k\pi\right)} | k \in \mathbb{Z} \}$

(C) $\{e^{\frac{\pi}{3}+2k\pi} | k \in \mathbb{Z} \}$

(D) $\{e^{-\left(\frac{\pi}{2}+2k\pi\right)} | k \in \mathbb{Z} \}$

21. The equation $|z - 1| + |z + 1| = 3$, where z is a complex number, represents

(A) an ellipse

(B) a circle

(C) a straight line

(D) a hyperbola

22. If a is an element of a group G, the order of a is n and p is relatively prime to n, then the order of a^p is

(A) 1

(B) pn

(C) n

(D) n^p

23. A common solution for the system of congruences

$x \equiv 2 \pmod 3$

$x \equiv 3 \pmod 5$

$x \equiv 2 \pmod 7$ is

(A) 5

(B) 8

(C) 58

(D) 23

24. If ϕ is Euler's ϕ-function and $\phi(n) \equiv 2 \pmod 4$, then n =

(A) 49

(B) 60

(C) 125

(D) 169

25. If F, K, L are fields such that L is a finite extension of F and K is a subfield of L which contains F, then

(A) $[L : F] \mid [K : F]$

(B) $[K : F] \mid [L : F]$

(C) $[L : F] \mid [L : K]$

(D) $[L : K] \mid [K : F]$

26. If the polynomial $x^4 + 1$ over F is an irreducible polynomial, then F =

(A) $\mathbb{Z}_2$

(B) $\mathbb{C}$

(C) $\mathbb{Z}_{13}$

(D) Q

27. If $\mathbb{R}$ denotes the field of real numbers, then $\frac{\mathbb{R}[x]}{(x^2+1)}$ is a field isomorphic to

(A) $\mathbb{R}$

(B) $\mathbb{C}$

(C) $\mathbb{R}(x)$

(D) $\mathbb{C}(x)$

28. The remainder when 7^{50007} is divided by 23 is

(A) 22

(B) 4

(C) 7

(D) 8

29. The order of a 2-Sylow subgroup in the symmetric group S_6 is

(A) 16

(B) 8

(C) 4

(D) 32

30. Which one of the following statements is true ?

(A) $\mathbb{R}$ in the lower limit topology is connected

(B) If A is any proper subset of $\mathbb{R}$ in the standard topology, then BdA = ϕ

(C) $\mathbb{R}$ in the finite complement topology is not a Hausdorff space

(D) $\mathbb{R}$ in the standard topology is not path connected

31. Let τ be the dictionary order topology on $\mathbb{R} \times \mathbb{R}$ and τ' be the product topology on $\mathbb{R}_d \times \mathbb{R}$, where $\mathbb{R}_d$ denotes $\mathbb{R}$ in the discrete topology. Then which one of the following statements is true ?

(A) $\tau = \tau'$

(B) τ and τ' are not comparable

(C) τ is strictly finer than τ'

(D) τ' is strictly finer than τ

32. If A is a subset of a topological space X, which one of the following statements is not true ?

(A) BdA = $\overline{A} \cap \overline{(X - A)}$

(B) Int A $\cap$ BdA $\neq \phi$

(C) $\overline{A}$ = Int A $\cup$ BdA

(D) BdA = ϕ if and only if A is both open and closed

33. Which one of the following statements is not true ?

(A) If X is connected, then for every non empty proper subset A of X, BdA $\neq \phi$

(B) The union of connected spaces that have a point in common is connected

(C) The image of a connected space under a continuous map is connected

(D) Every connected space is path connected

34. The smallest positive integer for which $\left(\frac{1+i}{1-i}\right)^n = 1$, holds is

(A) 3

(B) 4

(C) 2

(D) 8

35. Let G be a cyclic group of order 10. Then the number of elements of order 5 is

(A) 4

(B) 2

(C) 5

(D) 1

36. Which one of the following are the integral curves of the differential equations ?

$$\frac{dx}{cy - bz} = \frac{dy}{az - bx} = \frac{dz}{bx - ay} \text{ ?}$$

(A) straight lines

(B) circles

(C) parabolas

(D) ellipses

37. If n is the smallest integer such that $\frac{\phi(n)}{n} < \frac{1}{4}$, where ϕ is Euler's ϕ -function, then n =

(A) 210

(B) 2310

(C) 105

(D) 385

38. If y = x is a solution of the differential equation $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$, then the second linearly independent solution of the differential equation is

(A) x^{-1}

(B) x^2

(C) x^{-2}

(D) $x^n, n \in \mathbb{Z}^+$

39. The partial differential equation of the set of all right circular cones whose axes coincide with z-axis is

(A) $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$

(B) $y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}$

(C) $x^2 \frac{\partial^2 z}{\partial x^2} = y^2 \frac{\partial^2 z}{\partial y^2}$

(D) $y^2 \frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$

40. Suppose u(x, y) satisfies Laplace's equation $\Delta^2 u = 0$ in $\mathbb{R}^2$ and u = x on the unit circle. Then at the origin

(A) $u \to \infty$

(B) u attains a finite minimum

(C) u attains a finite maximum

(D) $u = 0$

41. The surface satisfying $\frac{\partial^2 u}{\partial y^2} = 3x^3 y$ containing two lines $y = 0 = u$ and $y = 1 = u$ is

(A) $u = \frac{x^3y^3}{2} + y\left(1 - \frac{x^3}{2}\right)$

(B) $u = -\frac{y^2}{8ax} + 1$

(C) u = sinx.cosy

(D) no such surface exists

42. Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 3 \\ 2 & 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 2 & 3 \\ 2 & 2 & 4 \\ 9 & 8 & 7 \end{pmatrix}$.

Consider the following statements

P_1 : The matrix A has a LU decomposition

P_2 : The matrix B has no LU decomposition.

Which one of the following statements is true ?

(A) Both P_1 and P_2 are true

(B) Only P_1 is true

(C) Only P_2 is true

(D) Neither P_1 nor P_2 is true

43. A solution of the Volterra integral equation

$$x^3 = \int_0^x (x-t)^2\, y(t)\, dt \text{ is}$$

(A) $y(x) = 0$

(B) $y(x) = 3$

(C) $y(x) = 2$

(D) $y(x) = 1$

44. For what values of m and n do the transformation equations

$Q = q^m \cos np$

$P = q^m \sin np$

represent a canonical transformation ?

(A) $m = \frac{1}{2}, n = 1$

(B) $m = 1, n = 2$

(C) $m = \frac{1}{2}, n = 2$

(D) $m = n = 1$

45. Extremals of

$$\iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 2zf(x, y) \right] dx\, dy$$

are solutions of

(A) Laplace equation

(B) Poisson's equation

(C) Wave equation

(D) Biharmonic equation

46. The sequence $\left\{A_n = \left[\frac{1}{n}, 1+\frac{1}{n}\right), n \geq 1\right\}$ converges to which one of the following ?

(A) [0, 1)

(B) (0, 1]

(C) (0, 1)

(D) [0, 1]

47. If $X_1, X_2, \ldots, X_n$ are independent and identically distributed Bernoulli random variables taking values 0 and 1 with probabilities ½ each and if $Y_n = X_1 + \ldots + X_n$, which one of the following is correct ?

(A) $E(Y_n) > V(Y_n)$

(B) $E(Y_n) < V(Y_n)$

(C) $E(Y_n) = V(Y_n)$

(D) $E^2(Y_n) = V(Y_n)$

48. In a linear programming problem with four constraints and six variables which of the following is equal to the maximum number of basic solutions ?

(A) 4

(B) 10

(C) 24

(D) 15

49. In the principle of ordinary least squares, which one of the following is true ?

(A) The sum of squares of deviations from actual observations is maximized

(B) The sum of squares of absolute deviations from actual values is minimized

(C) The sum of squares of deviations from actual values is minimized

(D) The sum of squares of absolute deviations from estimated values is minimized

50. The distribution of points scored by 100 students in a competition is known to be normal with mean 55.5 and standard deviation 3.6. What is the percentage of students scoring at least 55.5 points ?

(A) 50

(B) 60

(C) 40

(D) 70

51. If f(x) = $\frac{4}{\pi(1+x^2)}$, 0 < x < 1, and 0 otherwise, is the probability density function of a random variable X, then what is E(X) ?

(A) log 4

(B) $\frac{\log 4}{\pi}$

(C) π log 4

(D) $\frac{\pi}{\log 4}$

52. If random variable X has variance V(X) finite, which one of the following is true ?

(A) $P\left(|X - E(X)| < 2\sqrt{V(X)}\right) \geq \frac{1}{4}$

(B) $P\left(|X - E(X)| < 2\sqrt{V(X)}\right) \geq \frac{1}{2}$

(C) $P\left(|X - E(X)| < 2\sqrt{V(X)}\right) \geq \frac{3}{4}$

(D) $P\left(|X - E(X)| < 2\sqrt{V(X)}\right) \geq \frac{7}{8}$

53. If η is the characteristic function of a random variable X, which one of the following is not a characteristic function ?

(A) $\bar{\eta}$

(B) $\sqrt{\eta}$

(C) $\eta^{\frac{2}{3}}$

(D) 2η

54. If $P(X_n = 1) = \frac{1}{n^2} = 1 - P(X_n = 0), n \geq 1$, which of the following is equal to $P(\limsup_{n\to\infty} X_n = 1)$?

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{1}{e}$

55. If $P(X = k) = \frac{k}{15}$, k = 1, 2, 3, 4, 5, which of the following is equal to E(X | X > 1) ?

(A) 3

(B) 45

(C) $\frac{45}{14}$

(D) $\frac{45}{16}$

56. If (X, Y) has joint probability density function f(x, y) = 8xy, 0 < x < 1, 0 < y < x and 0 elsewhere, which one of the following is equal to E(X) ?

(A) $\frac{1}{5}$

(B) $\frac{2}{5}$

(C) $\frac{3}{5}$

(D) $\frac{4}{5}$

57. What is the moment generating function of uniform random variable over (0, 1) ?

(A) $\frac{\exp(t)-1}{t}$

(B) $\frac{1-\exp(t)}{t}$

(C) exp(t) – 1

(D) 1 – exp(t)

58. What is E(X) equal to if X has Beta (m, n) distribution ?

(A) mn

(B) $\frac{mn}{m+n}$

(C) $\frac{m+n}{mn}$

(D) $\frac{m}{m+n}$

59. If X and Y are independent and identically distributed Poisson random variables, which one of the following is the conditional distribution of X, given X + Y ?

(A) Poisson

(B) Binomial

(C) Multinomial

(D) Compound Poisson

60. Given that X has Binomial distribution with parameters n and $\frac{1}{2}$ and P(x = 4) = P(x = 5), which one of the following is true ?

(A) n = 7

(B) n = 8

(C) n = 9

(D) n = 10

61. If X_1, X_2,..., X_n are independent and identically distributed uniform (0, 1) random variables, what is the limit in distribution of $n(\max.\{X_1,...,X_n\} - 1)$?

(A) Normal (0, 1)

(B) Chi-square

(C) Fréchet

(D) Weibull

62. If Y_1, Y_2, Y_3 denote the order statistics from a random sample of size three from standard exponential distribution, with reference to $Y_3 - Y_2$ and Y_2, which of the following is true ?

(A) They are correlated

(B) They are identically distributed

(C) Both have Gamma distribution

(D) They are independent

63. What is the value of the objective function at an optimal solution of the linear programming problem ?

Minimize $x_1 + x_2$

subject to $x_1 - x_2 = -5$

$x_1 \geq 0, x_2 \geq 0$?

(A) 5

(B) – 5

(C) 10

(D) 0

64. Which one of the following is true for a 'two-person zero-sum' game ?

(A) the two persons have equal gains

(B) the gain of one player is equal to the loss of the other

(C) the gain of one player is equal to zero

(D) nothing can be said about gain and loss of the two persons

65. In a design of experiment with 5 factors each considered at 2 levels, the key block is given as : (I), BC, DE, BCDE, ABD, ACD, ABE, ACE. Which one of the following gives the confounded interactions ?

(A) ABC, ACE, BCDE

(B) ADE, ABCD, BCE

(C) ACE, ABD, BCDE

(D) ABC, ADE, DCBE

66. In a 2^3 – factorial experiment, principal blocks of replicates 1 and 2 respectively consist of {(I), A, BC, ABC} and {ABC, AC, B, (I) }. What are the confounded interaction effects in the two replicates respectively ?

(A) ABC and AB

(B) BC and AB

(C) AB and BC

(D) BC and AC

67. What is the error degrees of freedom in an RBD with 4 blocks, 6 treatments and when one missing value is estimated ?

(A) 14

(B) 15

(C) 23

(D) 24

68. What is Yates' coefficient of association in a 2 × 2 contingency table ?

(A) $\dfrac{ad-bc}{ad+bc}$

(B) $\dfrac{a-c}{b-d}$

(C) $\sqrt{\dfrac{\chi^2}{N+\chi^2}}$

(D) $\sqrt{\dfrac{N+\chi^2}{\chi^2}}$

69. Which of the following is the coefficient of mean square contingency ?

(A) $\sqrt{\dfrac{1+\chi^2}{\chi^2}}$

(B) $\sqrt{\dfrac{\chi^2}{1+\chi^2}}$

(C) $\sqrt{\dfrac{\chi^2}{N+\chi^2}}$

(D) $\sqrt{\dfrac{N+\chi^2}{\chi^2}}$

70. Which of the following is the moment estimator of β if X has gamma distribution with $E(X) = \alpha\beta$ and $V(X) = \alpha\beta^2$?

(A) $\dfrac{S}{\overline{X}}$

(B) $\dfrac{S^2}{\overline{X}}$

(C) $\dfrac{\overline{X}^2}{S}$

(D) $\dfrac{\overline{X}^2}{S^2}$

71. Which of the following is the MLE of $P(X_1 \geq 1)$, given that $\{X_1, X_2,...,X_n\}$ is a random sample from the probability density function $f(x\,;\,\theta) = \dfrac{1}{\theta}\exp\left(-\dfrac{x}{\theta}\right)$, x > 0 and 0 otherwise ?

(A) $\exp\left(-\overline{X}\right)$

(B) $1-\exp\left(-\overline{X}\right)$

(C) $\exp\left(-\dfrac{1}{\overline{X}}\right)$

(D) $1-\exp\left(-\dfrac{1}{\overline{X}}\right)$

72. Which of the following is the MVUE of θ from a random sample $\{X_1, X_2, X_3, X_4\}$ from uniform (0, θ) ?

(A) $\frac{5}{4}$ min. $\{X_1, X_2, X_3, X_4\}$

(B) $\frac{5}{4}$ max. $\{X_1, X_2, X_3, X_4\}$

(C) $\frac{4}{5}$ min. $\{X_1, X_2, X_3, X_4\}$

(D) $\frac{1}{4}\{X_1 + X_2 + X_3 + X_4\}$

73. What is the distribution of $2X_1^2 + 3X_2^2 - X_3^2 + X_4^2$ if (X_1, X_2, X_3, X_4) has a 4 – variate normal distribution ?

(A) Wishart

(B) 4-variate normal

(C) Hotelling's T^2

(D) Chi-square

74. What is a necessary and sufficient condition for $(X-\mu)' A (X-\mu)$ to have χ^2 (m) -distribution, given that X has a p-variate normal distribution with mean vector μ, non-singular variance-covariance matrix Σ and $m = \text{trace}(A\Sigma)$?

(A) $A\Sigma A = A$

(B) $A\Sigma = A$

(C) $\Sigma A = A$

(D) $\Sigma A \Sigma = \Sigma$

75. Which of the following is a sufficient statistic for θ, given that $\{X_1, X_2, ..., X_n\}$ is a random sample from the probability density function $f(x ; \theta) = \theta x^{\theta-1}, 0 < x < 1$, and 0 otherwise ?

(A) $\frac{X_1 + ... + X_n}{n}$

(B) $\prod_{i=1}^{n} X_i$

(C) max. $\{X_1,...,X_n\}$

(D) min. $\{X_1,...,X_n\}$

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