13.8.2019

Answer any <u>Six</u> Questions :   (6 x 5 =30 Marks)

1. For what values of k, the below set of equations have a solution. Solve them completely in each cases.

   $x + y + z = 1$

   $2x + y + 4z = k$

   $4x + y + 10z = k^2.$

2. Reduce the matrix A to its normal form and hence find its rank, where

$$A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}.$$

3. Find the nth derivative of the following:

   a) $\dfrac{1}{(x+2)(x+1)}$.   b) $e^x \sin x \sin 2x.$

4. If $y = e^{m \sin^{-1} x}$, prove that $(1-x^2)y_{n+2} + (2n+1)xy_{n+1} - (n^2 + m^2)y_n = 0.$

5. If $u = u(x,y)$ is a homogeneous function of degree n, then prove that

   a) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = nu.$

   b) Using (a) prove that $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} - n(n-1)u.$

6. If $x = \log_e\left(\dfrac{x^{5/2} + y^{5/2}}{x^{1/2} - y^{1/2}}\right)$, then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2.$

7. Find the symmetrical form of the line of intersection of the planes $2x + 3y + 5z - 1 = 0$ and $3x + y - z + 2 = 0.$

8. Find the bisector of the acute angle between the planes $2x - y + 2z + 3 = 0$ and $3x + 2y + 6z + 8 = 0.$