Register Number:

DATE: 26 - 11 - 2020

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
B.Sc. MATHEMATICS – III SEMESTER
SEMESTER EXAMINATION: NOVEMBER 2020

MT318 : MATHEMATICS-III

Time- 2 ½ hrs. **Max Marks-70**

This question paper has **TWO** printed pages and **FOUR** parts.

I. ANSWER ANY FIVE OF THE FOLLOWING. (5X2=10)

1) Define the order of an element. What is the order of i in the multiplicative group of fourth root of unity?
2) What are the generators of $(Z_6, \oplus_6)$?
3) Define group homomorphism. Is $\varphi : (R,+) \to (R,+)$, $\varphi(x) = e^x$ a group homomorphism?
4) Find all distinct cosets of $H = \{0,2,4\}$ in $(Z_6, \oplus_6)$.
5) Show that the set $\mathbb{N}$ has no limit point.
6) Evaluate $\lim\limits_{x \to 0} \frac{x - \sin x}{x^3}$.
7) Find the critical points of the function $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$
8) Solve $(D^3 - 3D^2 + 4)y = 0$

II. ANSWER ANY THREE OF THE FOLLOWING. (3X6=18)

9) Prove that $|a^{-1}| = |a|$, $\forall\, a \in G$ where order of a is finite.
10) Let G be a group and H be a subgroup of G then prove that
    a) $aH = bH$ if and only if $a \in bH \quad \forall a, b \in G$.
    b) $aH = Ha$ if and only if $aHa^{-1} = H$, $\forall\, a \in G$.
11) Prove that a subgroup H of G is normal in G if and only if $x\,H\,x^{-1} \subseteq H \quad \forall x \in G$
12) Let φ be a homomorphism from the group $(G, .)$ to $(\overline{G}, .)$ and let H be a subgroup of G, then prove the following.
    a) $\varphi(H) = \{\varphi(h) / h \in H\}$ is a subgroup of $\overline{G}$.
    b) If H is normal in G, then $\varphi(H)$ is normal in $\varphi(G)$.
13) State and prove the first theorem of isomorphism.

III. ANSWER ANY FOUR OF THE FOLLOWING. (4X6=24)

14) Prove that if a function f is continuous on a closed interval $[a, b]$, then it attains its bounds at least once in $[a, b]$.

15) State and Prove Cauchy mean value theorem.

16) Obtain the expansion of $\log(1 + e^x)$ up to term containing x^4.

17) Find the points of maxima and minima of the function $1+\sin(x^2 + y^2)$.

18) A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction using the method of Lagrange multipliers.

IV. ANSWER ANY THREEOF THE FOLLOWING. (3X6=18)

19) Solve the differential equation $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = e^x(1 + \sin x)$.

20) Solve the differential equation $x\frac{d^2y}{dx^2} - 2(x + 1)\frac{dy}{dx} + (x + 2)y = (x - 2)e^x, x \neq 0$ by finding the part of complementary function

21) Solve $\frac{d^2y}{dx^2} + 4\,cosec2x\frac{dy}{dx} + 2\tan^2 x\, y = e^x cot x$ by reducing it to normal form.

22) Show that the following equation is exact and hence solve

$(1 - x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0, x \neq 1$, given that $y(0) = 1, y'(0) = 0$.

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