14.8.2019

ST. JOSEPH'S COLLEGE(AUTONOMOUS), BANGALORE-27

B.Sc. MATHEMATICS – V SEMESTER

MID-SEMESTER TEST – AUGUST 2019

MT 5115 – MATHEMATICS V

Answer any Six out of Eight Questions : **(6 x 5 Marks=30 Marks)**

1. Prove that $grad(div\,\hat{r}) = \left(\frac{-2}{r^3}\right)\vec{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = |\vec{r}|$.

2. If $\vec{F}$ and $\vec{G}$ are vector point functions, prove that

$$curl(\vec{F}\times\vec{G}) = (div\,\vec{G})\vec{F} - (div\,\vec{F})\vec{G} + (\vec{G}\cdot\nabla)\vec{F} - (\vec{F}\cdot\nabla)\vec{G}.$$

3. Find constants a and b such that $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2y)\hat{k}$ is irrotational. Also find a scalar function φ such that $\vec{F} = \nabla\varphi$.

4. (i) In a ring $(R,+,\cdot)$, prove that $a\cdot 0 = 0$ and $a\cdot(-b) = -(a\cdot b), \forall a,b \in R$.

   (ii) Find the zero and unity of the commutative ring $(\mathbf{Q}, \oplus, \square)$, where $a \oplus b = a + b + 1$ and $a \,\square\, b = a + b + ab, \forall a,b \in \mathbf{Q}$. Show that this ring is an integral domain.

5. Define a subring. Prove that a non-empty subset S of a ring $(R,+,\cdot)$ is a subring of R if and only if $S + (-S) = S$ and $S\cdot S \subseteq S$.

6.Solve

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 20e^{-2x}$$

7.Solve

$$(5+2x)^2\frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 8(5+2x)^2$$

8.Solve

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = x^2$$