TIME: 1 HOUR

MAX. MARKS: 30

Answer any SIX questions of the following and each question carries FIVE marks.

1) Solve the variational problem $I = \int_1^2 x^2 (y')^2 \, dx$ given $y(1) = 1$, $y(2) = 1$.

2) Find the curve through $(0, 1)$ and $(1, 2)$ along which
   $I = \int_0^1 [y^2 - y\,y' + (y')^2] \, dx$ is a minimum.

3) Prove that the shortest arc joining the two points on a sphere is the minor arc of the great circle.

4) Evaluate $\int_C (x + y + z) \, ds$ where $'C'$ is the line joining the points $(1, 2, 3)$ and $(4, 5, 6)$.

5) Check whether the following line integral
   $\int_C (4xy - 3x^2 z^2) \, dx + (2\,x^2) \, dy - 2\,x^3 z \, dz$ is independent of path and hence
   evaluate.

6) Evaluate $\iint_R y\,dxdy$ where $'R'$ is the region bounded by $y^2 = 4x$ and $x^2 = 4y$.

7) Evaluate $\iint y \sqrt{x^2 + y^2} \, dxdy$ over the positive quadrant of the circle
   $x^2 + y^2 = a^2$, by changing into polar co-ordinates.

8) Find the surface area of the sphere $x^2 + y^2 + z^2 = 4$ by using double integral.