ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE -27
MID-SEMESTER TEST - AUGUST 2016
M.Sc. MATHEMATICS - I SEMESTER
MT 7114 : ALGEBRA-I

TIME: 90 min **MAX MARKS: 35**

Answer any seven of the following. **7 x 5 = 35**

1. Discuss the Dihedral group D_8.
2. State and prove the Second Isomorphism Theorem for groups.
3. Define inner automorphism of a group. Prove that the set I(G) of all inner automorphisms of a group G is isomorphic to G/Z(G).
4. If a finite group G acting on a set S, prove that for any $s \in S, |G| = |Orb_G(s)||Stab_G(s)|$. Hence prove that for any two subgroups H and K of G $|HK| = \frac{|H||K|}{|H \cap K|}$.
5. State and prove Cauchy-Frobenius theorem. Verify the theorem for the group $G = \{e, (132)(456)(78), (132)(456), (123)(456), (123)(456)(78), (78)\}$ acting on the set $S = \{1,2,3,4,5,6,7,8\}$.
6. Let H be a subgroup of a group G, and $S = \{gH \mid g \in G\}$. Define a homomorphism ψ of G into A(S) and show that kernel of ψ is the largest normal subgroup of G contained in H.
7. If G is a finite group, prove that $|G| = \sum|C(a)| = \sum\frac{|G|}{|N(a)|}$, where sum runs over one element *a* from each conjugate class. Verify this result for the symmetric group S_3.
8. Let G be a finite group of order p^n, where p is a prime. Then prove that the center of G, Z(G), is nontrivial and in particular when n = 2, Z(G) is equal to the group G.
9. State and prove Cauchy's theorem for finite groups.
10. Let G be a finite group of order $p^{\alpha}m$, where p is a prime and p does not divide m, then prove that there exists a Sylo p-subgroup of G.