ST.JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
MID-SEMESTER TEST - AUGUST 2016
M.Sc. MATHEMATICS- I SEMESTER
MT-7214 REAL ANALYSIS

TIME: 90 minutes **MAX. MARKS: 35**

Answer any 5 questions from the following. Each of the following questions carries 7 marks

1. Show that the function $f(x)=x$ is integrable over $[0,C]$ where $C\in\Re$ and that $\int_0^C f(x)\,dx=\frac{C^2}{2}$
2. If P^* is a refinement of P, then prove that $L(P,f,\alpha)\le L(P^*,f,\alpha)\le U(P^*,f,\alpha)\le U(P,f,\alpha)$
3. If $f\in\Re(\alpha)$ and $g\in\Re(\alpha)$ on $[a,b]$, then prove that $|f|\in\Re(\alpha)$ and $\left|\int_a^b f\,d\alpha\right|\le\int_a^b |f|\,d\alpha$
4. If $f\in\Re(\alpha_1)$ and $f\in\Re(\alpha_2)$ on $[a,b]$ then prove that $f\in\Re(\alpha_1+\alpha_2)$ and $\int_a^b fd(\alpha_1+\alpha_2)=\int_a^b fd(\alpha_1)+\int_a^b fd(\alpha_2)$
5. Give an example of a function f to show that $|f|\in\Re(\alpha)$ on $[a,b]$ but $f\notin\Re(\alpha)$ on $[a,b]$
6. Suppose F and G are differentiable functions on $[a,b]$, $F'=f\in\Re$ and $G'=g\in\Re$, then prove that $\int_a^b F(x)g(x)\,dx=F(b)G(b)-F(a)G(a)-\int_a^b G(x)f(x)\,dx$
7. If $f\in\Re$ on $[a,b]$, for $a\le x\le b$, put $F(x)=\int_a^x f(t)dt$, then prove that F is continuous on $[a,b]$; further more, if f is continuous at a point x_0 on $[a,b]$, then prove that F is differentiable at x_0, and $F'(x_0)=f(x_0)$.