ST.JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

MID-SEMESTER TEST – AUGUST 2016
M.Sc MATHEMATICS - I SEMESTER
MT 7314 - TOPOLOGY - I

Time: $1\frac{1}{2}$ hrs **Max marks-35**

Answer any SEVEN of the following questions 7 X 5 = 35

1. i) Define a Topological space. (2)
   ii) Give an example to show that the union of two topologies on X need not be a topology on X (3)

2. In a topological space show that
   i) The arbitrary intersection of closed sets is closed. (3)
   ii) Give an example of a nontrivial topology in which all the sets are both open and closed. (2)

3. Let (X, τ) be a topological space.
   i) Define a neighbourhood of a point. (2)
   ii) If A and B are neighbourhoods of a point x then prove that $A \cap B$ is also its neighbourhood. (3)

4. Let A be a subset of a topological space (X, τ).
   i) Define limit point of A and the derived set of A (1)
   ii) Prove that $d(\phi) = \phi$. (d(A) denotes the derived set of A.) (2)
   iii) If $A \subset B$ then $d(A) \subset d(B)$. (2)

5. Let A be a subset of a topological space (X, τ).
   i) Define interior of A (1)
   ii) If A^0 denotes the interior of A, prove that A is open if and only if $A^0 = A$ (4)

6. Let (X, τ) be a topological space.
   Prove that a point x belongs to the closure of A if and only if every open set G which contains x has a non empty intersection with A. (5)

7. Prove that $\bar{A} = A^0 \cup d(A)$ (5)

8. Let (X, τ) be a topological space.
   i) Define boundary of a set. (2)
   ii) If b(A) denotes the boundary of a set A then prove that $b(A) = \phi$ if and only if A is both open and closed. (3)

9. Prove that $f : X \to Y$ is continuous iff inverses of open sets of Y are open in X. (5)

10. Prove that $f : X \to Y$ is continuous iff $f(\bar{A}) \subset \overline{f(A)}$ (5)