St. Joseph's College (Autonomous), Bangalore
Mid Semester Test - August 2016
M.Sc. Mathematics – III Semester
Differential Geometry MT-9214

Time: 1 ½ hours **Max. Marks: 35**

Answer any 5 questions. **(5x7=35)**

1. Let f and g be functions on R^3, v_p and w_p tangent vectors, a and b numbers. Then
   a) $(av_p + bw_p)[f] = av_p[f] + bw_p[f]$
   b) $v_p[af + bg] = av_p[f] + bv_p[g]$
   c) $v_p[fg] = v_p[f] \cdot g(p) + f(p) \cdot v_p[g]$
2. Evaluate the 1-form $\phi = x^2 dx - y^2 dz$ on the vector fields
   a) $V = xU_1 + yU_2 + zU_3$
   b) $W = xy(U_1 - U_3) + yz(U_1 - U_2)$
   c) $\frac{1}{x}V + \frac{1}{y}W$
3. Let f and g be functions, ϕ and ψ 1-forms. Then
   a) $d(fg) = df\ g + f\ dg$
   b) $d(f\phi) = df \wedge \phi + f \wedge d\phi$
   c) $d(\phi \wedge \psi) = d\phi \wedge \psi - \phi \wedge d\psi$
4. Define unit speed curve and plane curve. Let β be a unit speed curve in R^3 with $\kappa > 0$. Then β is a plane curve if and only if $\tau = 0$.
5. Let α be a regular curve in R^3. Then
   a) $T = \dfrac{\alpha'}{\| \alpha' \|}$
   b) $N = B \times T$
   c) $B = \dfrac{\alpha' \times \alpha''}{\| \alpha' \times \alpha'' \|}$
   d) $\kappa = \dfrac{\| \alpha' \times \alpha'' \|}{\| \alpha' \|^3}$
   e) $\tau = \dfrac{(\alpha' \times \alpha'') \cdot \alpha'''}{\| \alpha' \times \alpha'' \|^2}$
6. Derive the cylindrical and spherical frame field on R^3.
7. For any function f, show that

$$A = \begin{pmatrix} \cos^2 f & \cos f \sin f & \sin f \\ \sin f \cos f & \sin^2 f & -\cos f \\ -\sin f & \cos f & 0 \end{pmatrix}$$

is the attitude matrix of a frame field, and compute its connection forms.