ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. Physics – SEMESTER III

MID-SEMESTER TEST - August 2016

PH 9115: QUANTUM MECHANICS-II

Time:1hr. 30 min. **Maximum Marks: 35**

This question paper has 3 printed pages and 2 parts.

PART A **MAX. MARKS 2x10=20**

Answer any TWO full questions.

1.

a) Starting with the polar part of the Schrodinger equation (actually the sourceless Laplace equation – with applications, therefore, in other fields as well):

$$\sin\theta\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\left[l(l+1)\sin^2\theta-m^2\right]\Theta=0$$

and making the variable change $x=\cos\theta$ obtain the Associated Legendre Equation

$$\frac{d}{dx}\left[(1-x^2)\frac{d\Theta}{dx}\right]+\left[l(l+1)-\frac{m^2}{1-x^2}\right]\Theta=0$$ **(8 Marks)**

b) For the azimuthal part of the Schrodinger equation, we have $\frac{d^2\Phi}{d\varphi^2}+m^2\Phi=0$. What is the solution to this differential equations. Provide a unified solution that accepts positive and negative values of m . **(2 Marks)**

2.

a) Using the classical relation: $\vec{L}=\vec{r}\,x\,\vec{p}$ obtain the quantum mechanical operators $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ in cartesian coordinates. **(2 Marks)**

b) The quantum mechanical operator for the square of the angular momentum is defined as $\hat{L}^2=\hat{L}_x^2+\hat{L}_y^2+\hat{L}_z^2$ and that the angular momentum ladder operators are defined as $\hat{L}_\pm=\hat{L}_x\pm i\hat{L}_y$. The eigenvalue equations satisfied by $\hat{L}^2$ and $\hat{L}_z$ for an eigenstate $|nlm\rangle$ are given as: $\hat{L}^2|nlm\rangle=\hbar^2 l(l+1)|nlm\rangle$ and $\hat{L}_z|nlm\rangle=\hbar m|nlm\rangle$. With respect to this eigenstate, show that

- $\langle L_x\rangle=\langle L_y\rangle=0$ **(4 Marks)**
- $\langle L_x^2\rangle=\langle L_y^2\rangle=\frac{\hbar^2 l(l+1)-\hbar^2 m^2}{2}$ **(4 Marks)**

2h

3.

a) Consider a two particle system described by a wavefunction: $\psi(r_1, r_2, t)$ satisfying the time dependent Schrodinger equation: $i\hbar\frac{\partial\psi}{\partial t}=H\psi=E\psi$. Where $H=-\frac{\hbar^2}{2m_1}\nabla_1^2-\frac{\hbar^2}{2m_2}\nabla_2^2+V(r_1,r_2,t)$ is the Hamiltonian describing the two particle system. If the potential is time-independent, show that $\psi(r_1, r_2,t)=\psi(r_1,r_2)e^{-\frac{i}{\hbar}Et}$.

(4 Marks)

b) For the stationary wave function for a two particle system: $\psi(r_1, r_2)$ define the exchange operator $\hat{P}_{ex}\psi(r_1,r_2)=\psi(r_2,r_1)$.

- Obtain the eigenvalues of the operator $\hat{P}_{ex}$ **(2 Marks)**
- Show that the eigenfunctions of this operator represent two distinct kind of particles. **(4 Marks)**

PART B **MAX. MARKS 3x5=15**

Answer any THREE full questions.

[Constants: h=6.6x10^{-34} J s (Planck's constant), 1eV = 1.6x10^{-19} J (electron volt to Joules), c=2.99x10^{8} m/s (speed of light),1Å = 1x10^{-10}m (Angstrom to meters), e = 1.6x10^{-19} C (electronic charge), m_{proton}=1.673x10^{-27}kg (mass of proton), $m_{electron}$=9.109x10^{-31}kg (mass of electron)]

4. Consider two non-interacting fermions in a 1-D Simple Harmonic Potential (ignore the spin effects for the moment). Write down the ground state energy wavefunction for the system and the corresponding energy eigenvalue . The expression for the wavefunction of a simple harmonic oscillator is given as $\psi_n(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}H_n(\xi)e^{-\xi^2/2}$ where $H_n(\xi)$ are the Hermite polynomials and $\xi=\sqrt{\frac{m\omega}{\hbar}}x$. The first five Hermite Polynomials are: $H_0=1$, $H_1=2\xi$, $H_2=4\xi^2-2$, $H_3=8\xi^3-12\xi$, $H_4=16\xi^4-48\xi^2+12$ **(5 Marks)**

5. An electron is in the spin state $\chi=A\begin{pmatrix}1-2i\\2\end{pmatrix}$

a) Determine the constant A by normalising χ **(1 Mark)**

b) If you measured $\hat{S}_z$ on this electron, what values could you get and what is the probability of each? What is the expectation value of $\hat{S}_z$? **(2 Marks)**

c) If you measured $\hat{S}_x$ on this electron, what values could you get and what is the probability of each? What is the expectation value of $\hat{S}_x$? **(2 Marks)**

6. Consider a particle on a 1-D ring (no forces act on the particle; the potential is zero). The particle is constrained to move on the ring so only the polar angle varies (you can assume the ring to be in the X-Y plane). The Laplacian in cylindrical polar coordinates is given by: $\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \psi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \psi}{\partial \varphi^2} + \frac{\partial^2 \psi}{\partial z^2}$. Solve the time-independent Schrodinger Equation subject to the boundary condition that $\psi(\varphi) = \psi(\varphi + 2\pi)$. Discuss the similarity to the azimuthal component of Schrodinger Equation in 3 dimensional spherical polar coordinates. **(5 Marks)**

7. For an angular momentum state of $j = \frac{3}{2}$ work out the matrix representing the $\hat{J}_x$ operator. The ladder operators are: $\hat{J}_{\pm} = \hat{J}_x + i\hat{J}_y$ and when operated by the ladder operators, a general ket $|jm\rangle$ transforms as $\hat{J}_{\pm}|jm\rangle = \hbar\sqrt{j(j+1) - m(m\pm 1)}|jm\pm 1\rangle$ **(5 Marks)**