Registration Number:

Date & Session

# ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU -27
## B.SC – V SEMESTER
## SEMESTER EXAMINATION: OCTOBER 2022
**(Examination conducted in December 2022)**

## MT5218 : MATHEMATICS - VI

**Time: 2 ½ Hours** **Max Marks: 70**

**This paper contains TWO printed pages and THREE parts.**

**I. Answer any FIVE of the following questions.** **(5X2=10)**

1. Find the real and imaginary part of the complex function $f(z)=\frac{\bar{z}}{z},\ z\neq 0.$
2. Evaluate $\lim_{z\to i}\left(\frac{z^2+1}{z^6+1}\right).$
3. Check whether the function $u=x^3-3xy^2$ is harmonic or not.
4. Evaluate $\oint_C \frac{\sin\pi z}{z-\pi}dz$ where $C$ is the circle $C:|z+2|=1.$
5. If $\vec{F}=3yz\hat{i}+2x^2\hat{j}+5xy\hat{k},$ then find curl $\vec{F}$ .
6. Find the unit normal to the surface $xyz=c$ at the point $(-1,2,3).$
7. Show that the vector field $\vec{F}=(x+3y)\hat{i}+(y-3z)\hat{j}+(x-2z)\hat{k}$ is solenoidal.
8. If $\vec{g}=x^2y\hat{i}+y^2z\hat{j}+z^2x\hat{k}$ then find $\nabla^2\vec{g}$.

**II. Answer any SEVEN of the following questions.** **(7X6=42)**

9. If $\frac{z-i}{z-1}$ is purely imaginary then show that its locus is a circle. Also find its radius and centre.
10. Show that the function $f(z)=\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ for $z\neq 0$ and $f(0)=0$ is continuous at the origin.
11. Find the bilinear transformation which maps $1,-i,-1$ in the $z-$plane onto $0,i,\infty$ in the $w-$plane respectively. Also find its fixed points.
12. State and prove the necessary condition that $f(z)=u+iv$ to be analytic in a domain $D$.

13. If $f(z) = u + iv$ is analytic, then show that $\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$.

14. Determine the analytic function $f(z)$, whose real part is $\left(r + \frac{1}{r}\right)\cos\theta,\ r \neq 0.$

15. State and prove Cauchy's integral theorem for analytic functions.

16. Evaluate $\oint_C \frac{e^{2z}}{(z+1)(z-2)}\, dz$ where $C$ is the circle given by $|z| = 3.$

17. Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}\, dz$ where $C$ is the circle given by $|z| = 1.5.$

**III. Answer any THREE of the following questions. (3X6=18)**

18. Find the directional derivative of $\phi(x, y, z) = x^2 - 2y^2 + 4z^2$ at the point $(1,1,-1)$ in the direction of $2\hat{i} + \hat{j} - \hat{k}$.

19. Find the constants $a$ and $b$ such that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2 y + z^3 = 4$ are orthogonal at the point $(1, -1, 2)$.

20. Prove that $div(r^n \vec{r}) = (n+3) r^n$. Hence show that $\frac{\vec{r}}{r^3}$ is solenoidal.

21. Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is a conservative force field and find its scalar potential.

22. For any vector field $\vec{f}$, Prove that $\nabla \times (\nabla \times \vec{f}) = \nabla(\nabla . \vec{f}) - \nabla^2 \vec{f}$.

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