Registration Number:
Date & session:

ST.JOSEPH'S UNIVERSITY, BENGALURU -27

M.Sc (PHYSICS) – I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2022
(Examination conducted in December 2022)
PH 7121 – CLASSICAL MECHANICS

Time: 2 Hours **Max Marks: 50**

This paper contains 3 printed pages and 2 parts

PART A

Answer any FIVE full questions. **(5x7=35)**

1. For a one-particle system, the Kinetic Energy is defined as $T=\frac{1}{2}m\dot{x}^2$ where m is the mass of the particle and x is its position.
   i. Show that if the Kinetic Energy of the particle is due to an externally applied force $F(x)$, then $T=\int F(x)dx$.
   ii. A force that depends only on position of the particle is a conservative force and $F(x)=-\frac{dV}{dx}$ where V is the potential energy of the particle. From (i) above and the force equation here, show that $T+V=\text{constant}$.
   iii. What is the significance of $T+V=\text{constant}$? [2+3+2]

2. A system described by a set of generalized coordinates q_k undergoes a change dq_k due to translation.
   (a) Show that the generalized force $Q_k=\sum_{i=1}^{N}\vec{F}_i\cdot\frac{\partial\vec{r}_i}{\partial q_k}$ is equal to the net force along the direction of translation.
   (b) Show also that the generalized momentum p_k is equal to the net linear momentum along the direction of translation. [3+4]

3. A particle of reduced mass μ is moving in a central force described by an inverse square law: $F(r)=-\frac{k}{r^2}$, with k being a positive constant (attractive force).
   (a) Obtain the expression for the "effective potential".
   (b) If the total energy of the particle is equal to the effective potential, obtain the expression for the distance of closest approach to the potential. What is its implication? [5+2]

4. If the Lagrangian explicitly depends on time, show that we can define an energy function $h(q,\dot{q},t)$ such that $\frac{dh}{dt}+\frac{\partial L}{\partial t}=0$. What is the implication for the energy function if the Lagrangian is not explicitly dependent on time?

5. Consider a system made up of N particles, constrained to positions described by vectors $\vec{r}_i(q_k)$, with q_k being the generalized coordinates corresponding to n degrees of freedom. If a rotation through dq_k is made about some arbitrary axis $\hat{n}$ show that the direction of the generalized momentum is the same as that of the net angular momentum of the system.

6. The waves on a string are represented by the wave equation: $\frac{\partial^2 y}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$. By introducing the variables $u=x-ct$ and $v=x+ct$ and some algebra, we can obtain the d'Alembert solution to the wave equation by envisioning the wave as a right moving component f and a left moving component g of the disturbance so that the general solution would be : $y(x,t)=f(x-ct)+g(x+ct)$. Using this solution for a **semi-infinite string**, obtain the two possibilities of reflection at a hard boundary at $x=0$.

7. For a rotating rigid body (with an angular velocity $\vec{\boldsymbol{\omega}}$), it can be shown that any vector $\vec{\boldsymbol{A}}$ representing a point in its interior measured with respect to its center of mass (or origin of the axis of rotation), transforms to inertial frame as: $\left(\frac{d\vec{\boldsymbol{A}}}{dt}\right)_{inertial}=\left(\frac{d\vec{\boldsymbol{A}}}{dt}\right)_{rot}+\vec{\boldsymbol{\omega}}\times\vec{\boldsymbol{A}}$, i.e. we can conceive of a new operator $\left(\frac{d}{dt}\right)_{inertial}=\left(\frac{d}{dt}\right)_{rot}+\vec{\boldsymbol{\omega}}\times$. Apply this operator on the position vector $\vec{\boldsymbol{r}}$ to obtain the transformation rule for velocity $\vec{\boldsymbol{v}}$ (explain each term).

PART-B

Answer any <u>THREE</u> full questions **(3x5=15)**

8. A bead of mass m is sliding down a wire kept at an angle α with respect to the horizontal. Show that the virtual work done on the bead by the constraint forces vanish.

9. Using the Euler equation find the extremum of the following functional: $J=\int_a^b\left(3x+\sqrt{\frac{\partial y}{\partial x}}\right)dx$.

10. The Lagrangian for the Simple Harmonic Oscillator is given as: $L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}m\omega^2x^2$ where x is the generalized coordinate and $\dot{x}$ is the generalized velocity. Compute the Hamiltonian of this system.

11. The semi-major axis of Neptune's orbit around the Sun is 4.495×10^{12} m. With the solar mass being: 1.99×10^{30} kg and the gravitation constant being: $6.674\times10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$. Using Kepler's Third law compute the period of revolution of Neptune around the Sun in Earth Years.

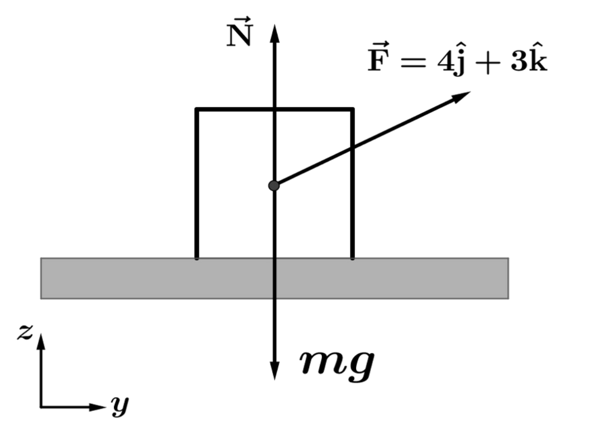

Fig. 1: Block on a table (Question 12)

12. A block is pulled on a frictionless table (Fig. 1). The motion is constrained to be along the horizontal direction (there is no motion along the vertical direction – see Fig. 1 to identify the vertical and horizontal directions). The applied force is given by the vector $\vec{F}=4\hat{j}+3\hat{k}$. Identify the constraint force and show that the virtual work due to the constraint force is equal to zero.