Register number:

Date and session:

ST. JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc (MATHEMATICS) - II SEMESTER
SEMESTER EXAMINATION: APRIL 2023
(Examination conducted in May 2023)
MT 8521- TOPOLOGY

(For current batch students only)

Time 2 Hours | Max Marks: 50

This paper contains TWO printed page and ONE part.
In question 3 answer either part a) or answer parts b) and c).

I. ANSWER ANY **FIVE** OF THE FOLLOWING.

1. a) Let $Y = (-1, 1)$ be given the subspace topology from $\mathbb{R}$. Which of the following are open in Y?

   i) $\{x : 0 \leq |x| < 1\}$ ii) $\{x : 1/2 \leq |x| < 1\}$

   Give reasons for your answers. **(4m)**

   b) Let X be a topological space. Prove that the following conditions hold:
   - i) X and ϕ are closed.
   - ii) Arbitrary intersection of closed sets are closed.
   - iii) Finite union of closed sets are closed. **(6m)**

2. a) Define closure and interior of a set. If $X = \{p, q, r, s\}$ and $\tau = \{X, \phi, \{p\}, \{p, q\}, \{s\}, \{p, s\}, \{p, q, s\}\}$. Find:

   i) int(A) if $A = \{p, q, r\}$. ii) cl(B) if $B = \{s\}$. **(5m)**

   b) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 5x + 4$ is a homeomorphism. **(3m)**

   c) If A is a subspace of X, prove that $j : A \to X$ is continuous. **(2m)**

3. a) Let X and Y be topological spaces. Let $f : X \to Y$. Prove that the following are equivalent:
   1) f is continuous.
   2) For every subset A of X, one has$f(\overline{A}) \subset \overline{f(A)}$.
   3) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X. **(10m)**

OR

   b) Show that the set of all invertible $n \times n$ matrices is not a connected subset of the set of all $n \times n$ matrices. **(3m)**
   c) Prove that the union of a collection of connected sets of X that have a common point is connected. **(7m)**
4. a) Show that $(0, 1)$ is not homeomorphic to $[0, 1)$. **(3m)**
   b) Prove that an open connected subset U of $\mathbb{R}^n$ is path connected. **(7m)**
5. a) Consider $X = \mathbb{R}$ with the standard topology. Check if the following are a cover for $\mathbb{R}$ or not.

   i. $A = \{(n, n + 1)\}, n \in \mathbb{Z}$.   ii. $B = \{[n, n + 1]\}, n \in \mathbb{Z}$.

   Give reasons for your answer. **(4m)**
   b) Prove that the image of a compact space under a continuous map is compact. **(6m)**
6. State and prove Lebesgue number lemma. **(10m)**
7. a) Let X be a toplogical space. Let one point sets in X be closed. Prove that X is normal if and only if given a closed set A and an open set U containing A, there is an open set V containing A such that $\bar{V} \subset U$. **(5m)**
   b) Prove that every compact Hausdorff space is normal. **(5m)**