Registration Number:

Date & Session:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU -27

M.Sc. MATHEMATICS – IV SEMESTER

SEMESTER EXAMINATION: APRIL 2023

(Examination conducted in May 2023)

MT0422: FINITE ELEMENT METHODS

(For current year students only)

Time: 2 ½ Hours **Max Marks: 70**

This paper contains THREE printed pages.

Answer any SEVEN full questions of the following: (7 X 10 =70 Marks)

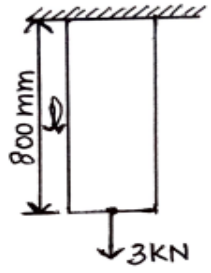
1. What is Finite element method? Write any five advantages, disadvantages and limitations of the method? [10M]

2. Given a differential equation $\frac{d^2u}{dx^2} - u + x = 0,\ 0 \le x \le 1$ with boundary conditions $u(0) = 0,\ u(1) = 0$, the corresponding functional to be extrimized is $I = \frac{1}{2}\int_0^1 \left[\left(\frac{du}{dx} \right)^2 - 2xu + u^2 \right] dx$. Find the solution to the differential equation using Rayleigh Ritz's method. [10M]

3. Given a differential equation $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0,\ 0 \le x \le 1$ with boundary conditions $y(0) = 1,\ y'(1) = 0.1$. Find $y(0.2)$ by using Galerkin's method. [10M]

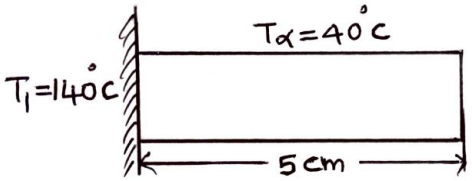
4. Derive the governing differential equation for one dimensional heat conduction in steady state. [10M]

5. A steel bar of 800 mm is subjected to the axial load of 3KN as shown in figure. Find the elongation of the bar by discretizing into two elements.
Data: $E = 2 \times 10^5\ N/mm^2,\ A = 300 mm^2$.



[10M]

6. Find the temperature distribution in the 1-D fin shown in figure. Take two elements for idealization.



Data: Specified Temperature $T_1 = 140^\circ C$, length $l = 5cm \Rightarrow l_1 = 2.5cm,\ l_2 = 2.5cm.$ Assume that the convection takes place throughout the surface and the surrounding temperature is $T_\alpha = 40^\circ C.$ Thermal conductivity $k = 70W/cmK$, convection heat transfer coefficient $h = 5W/cm^2\ K$, area $A = 3.14cm^2$, perimeter $p = 6.28cm$. [10M]

7. For the brick wall shown in figure, the inside temperature is $28^\circ C$ and outside of the wall is exposed to cold air at $-15^\circ C$. Determine the nodal temperature distribution in a steady state , by considering two 1- dimensional heat flow elements.

Data: $l = 0.3m \Rightarrow l_1 = 0.15m,\ l_2 = 0.15m,\ k_1 = k_2 = 0.7W/m^\circ C, h = 40W/m^{2\,\circ} C,\ A = 1m^2$

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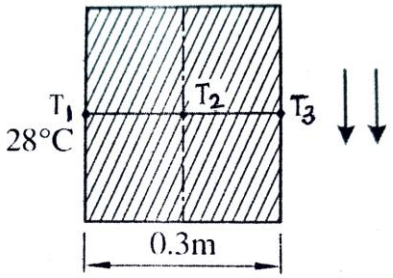
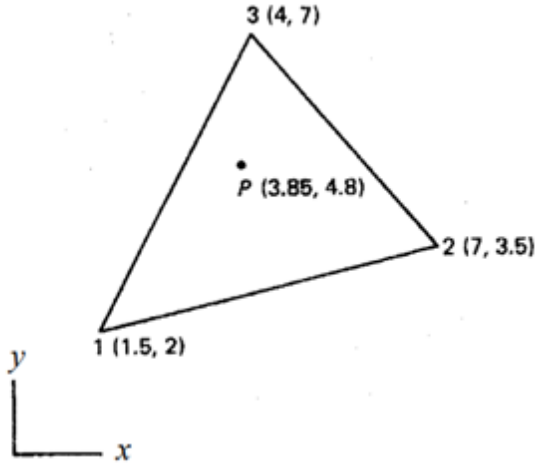


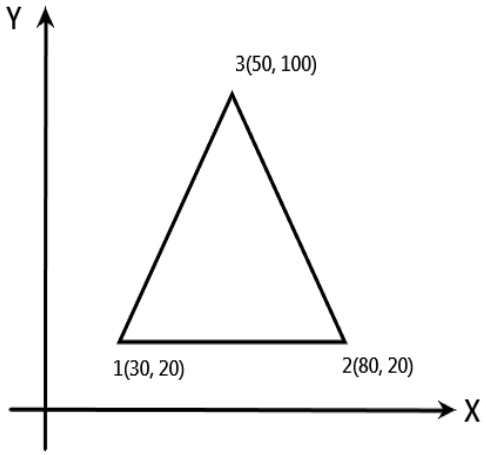
[10M]

8. Evaluate the shape functions N_1, N_2 and N_3 the interior point p for the rectangular element.



[10M]

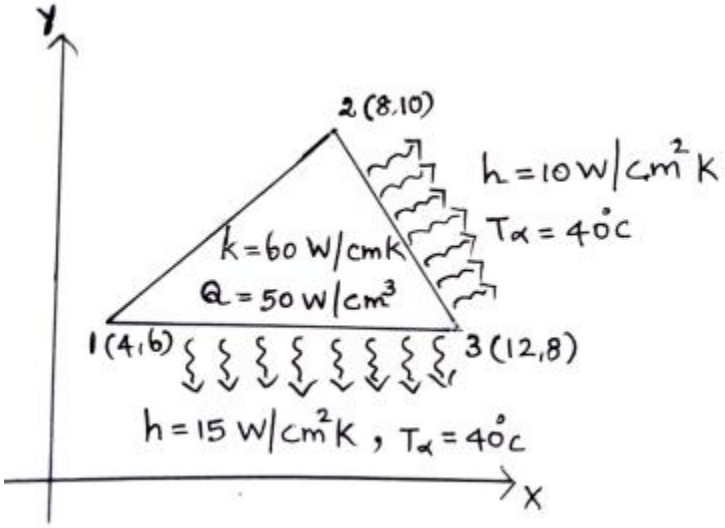
9. For the plane stress element shown in figure, evaluate the stiffness matrix . Assume the modulus of elasticity $E = 210 \times 10^3 N/mm^2$, Poisson's ratio $\mu = 0.25$ and element thickness $t = 10mm$. The coordinates are given in millimeters.



[10M]

(OR)

Compute the element matrices and force vectors for the element shown in figure, when the edges 2-3 and 3-1 experience convection heat loss.



[10M]