ST JOSEPH'S UNIVERSITY, BENGALURU-27
B.Sc (MATHEMATICS) - II SEMESTER
SEMESTER EXAMINATION: APRIL, 2023
(Examination conducted in May 2023)
MTOE 5 – MATHEMATICS FOR PHYSICAL SCIENCES
(For current batch second semester students only)

Time: 2 Hours   **Max. Marks:** 60

This paper contains **TWO** printed pages and **THREE** parts.

PART-A
ANSWER ANY <u>SIX</u> OF THE FOLLOWING   **(6×2=12)**

1. Find the order and degree of $\dfrac{d^2y}{dx^2} = -a^2 x$.

2. Check the exactness of the equation $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$.

3. Find $\dfrac{\partial u}{\partial x}$ of the function $u(x, y) = \dfrac{xy}{x + y}$.

4. If $u = 2x - 3y$, $v = 5x + 4y$, find $\dfrac{\partial(u, v)}{\partial(x, y)}$.

5. Find the critical points of the function $u(x, y) = x^2 + 5y^2 - 6x + 10y + 12$.

6. Prove that $L[e^{at}] = \dfrac{1}{s - a}$.

7. Find the Laplace transform of $[5^t + 2]$.

8. Find the inverse Laplace transform of $\left[\dfrac{4s - 1}{s^2 + 25}\right]$.

PART-B
ANSWER ANY <u>THREE</u> OF THE FOLLOWING   **(3×6=18)**

9. Solve $xy\,\dfrac{dy}{dx} = y + 2$.

10. Solve $\dfrac{dy}{dx} + y\,cotx = 4x\,cosecx$.

11. Solve $x\dfrac{dy}{dx} + (1 - x)y = x^2 y^2$.

12. Find the Laplace transform of $2^{t+1}sinh3t$.

13. Find the Laplace transform of $cost\ cos2t\ cos3t$.

ANSWER ANY <u>FIVE</u> OF THE FOLLOWING $\qquad$ **(5×6=30)**

14. Test the exactness and hence solve $(x^2 - ay)dx + (y^2 - ax)dy = 0$.

15. Verify Euler's theorem for $u(x, y) = x^3 - 2x^2y + 3xy^2 + y^3$.

16. Find $\dfrac{du}{dt}$ if $u = xy^2 + x^2y$, where $x = at^2$ and $y = 2at$.

17. Find the Taylor's series expansion of $f(x, y) = log(1 + x + y)$ at $x = y = 0$.

18. Find the inverse Laplace transform of $\left[\dfrac{s + 5}{(s^2 - 6s + 13)}\right]$.

19. Verify convolution theorem for the functions $f(t) = sint$ and $g(t) = e^{-t}$.

20. Solve by using Laplace transform: $\dfrac{d^2y}{dt^2} + k^2y = 0$ where k is a constant, given that $y(0) = 2, y'(0) = 0$.