Registration Number:

Date & session:

ST JOSEPH'S COLLEGE(AUTONOMOUS), BENGALURU-27
B.Sc (MATHEMATICS) - IV SEMESTER
SEMESTER EXAMINATION: APRIL, 2023
(Examination conducted in May 2023)
MTOE 5 – MATHEMATICS FOR PHYSICAL SCIENCES
(For current batch fourth semester students only)

Time: 2 Hours **Max. Marks:** 60

This paper contains **TWO** printed pages and **THREE** parts.

PART-A

ANSWER ANY SIX OF THE FOLLOWING **(6×2=12)**

1. Find the order and degree of $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^4 = e^{4x}$.
2. Check the exactness of the equation $(x^2 - ay)dx + (y^2 - ax)dy = 0$.
3. Find $\frac{\partial u}{\partial x}$ of the function $u(x, y) = sin\left(\frac{x}{y}\right)$.
4. If $u = 3x + 5y,\ v = 4x - 3y$, find $\frac{\partial(u, v)}{\partial(x, y)}$.
5. Find the critical points of the function $u(x, y) = x^3 + y^3 - 3xy$.
6. Prove that $L[a] = \frac{a}{s}$.
7. Find the Laplace transform of $2^t + 3$.
8. Find the inverse Laplace transform of $\left[\frac{1}{(s-4)^3}\right]$.

PART-B

ANSWER ANY THREE OF THE FOLLOWING **(3×6=18)**

9. Solve $x^2y\ \frac{dy}{dx} = y + 1$.
10. Solve $x\frac{dy}{dx} - 2y = 2x$.
11. Solve $x\frac{dy}{dx} + (1 - x)y = x^2y^2$.
12. Find the Laplace transform of $[(t + 2)^2e^t]$.
13. Find the Laplace transform of $sint\ sin3t\ sin5t$

PART-C

ANSWER ANY FIVE OF THE FOLLOWING **(5×6=30)**

14. Test the exactness and hence solve $(ax + hy + g)dx + (hx + by + f)dy = 0$.

15. Verify Euler's theorem for $u(x, y) = x^3 + y^3 + 3x^2y$.

16. Find $\frac{du}{dx}$ if $u = x^2 + xy + y^2$, and $y = sinx$.

17. Find the Taylor's series expansion of $f(x, y) = e^x\ log(1 + y)$ at $x = y = 0$.

18. Find the inverse Laplace Transform of $\frac{3}{2}\left[\frac{s^4 - 2s^2 + 1}{s^5}\right]$.

19. Verify convolution theorem for the functions $f(t) = 1$ and $g(t) = sint$

20. Solve by using Laplace transform: $\frac{d^2y}{dt^2} + k^2y = 0$ where k is a constant, given that $y(0) = 2, y'(0) = 0$.