Registration Number:

Date & Session

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU -27

B. Sc. (Statistics)– IV SEMESTER

SEMESTER EXAMINATION: APRIL 2023

(Examination conducted in May 2023)

ST 422 – Statistical Inference I

(For current batch students only)

Time: 2 Hours **Max Marks: 60**

This paper contains TWO printed pages and THREE parts

PART – A

I. Answer any FIVE of the following **3 X 5 = 15**

1. Define location – scale family with one example.
2. What is the difference between estimate and estimators?
3. State Neymann – Factorization Theorem.
4. If $X \sim U(0, \theta)$, derive the maximum likelihood estimator (MLE) for the parameter.
5. Give the procedure of construction of confidence interval using pivotal quantity method.
6. Define Simple hypothesis and Composite Hypothesis with example.
7. Explain the concept of acceptance region and rejection region with the help of graph.

PART – B

II. Answer any FIVE of the following **5 X 5 = 25**

8. Define consistency. State the sufficient conditions for consistency. Show that the sample variance, $s_n^2 = \frac{1}{n}\sum_i^n (x - \bar{x})^2$ is consistent with the population variance, σ^2 of a normal population.
9. Obtain the MVUE for σ^2 in the normal population $N(\mu_0, \sigma^2)$.
10. Find the MLE for the parameter λ of a Poisson distribution on the basis of sample of size n. Also obtain the MLE of $5\lambda^3 + 3\lambda^2 - 7\lambda + 8$.
11. Derive the confidence interval for difference in proportion with usual notations.
12. Derive the confidence interval for population variance when mean is unknown.
13. Let p be the probability that a coin will fall, head in single toss. In order to test the hypothesis $H_0: p = \frac{1}{2}$. The coin is tossed 6 times and H_0 is rejected if more than 4 heads are obtained. Find the size of the test. If $H_1: P = \frac{1}{3}$ find the power of the test.
14. Write a note on different types of tests based on critical region.

PART – C

III. Answer any TWO of the following **10 X 2 = 20**

15. A) Let x_1, x_2, x_3 be a random sample of size n = 3 taken from a normal population with mean μ and variance σ^2. Also let $\hat{\mu}_1 = \frac{x_1+2x_2+3x_3}{6}$ and $\hat{\mu}_2 = \frac{x_1+x_2+x_3}{3}$ be two estimators of μ. (a) Are $\hat{\mu}_1$ and $\hat{\mu}_2$ unbiased? (b) Compare the efficiency of $\hat{\mu}_1$ with respect to $\hat{\mu}_2$.

    B) Let x_1, x_2 be independent observations from a Poisson distribution with mean α. Show that $x_1 + x_2$ is sufficient for α. (6+4)

16. A) Let $x_1, x_2, \dots x_n$ be identically independently distributed $B(m, p)$ random variables, where m and p are unknown. Obtain moment estimators for both m and p.

    B) Derive 100(1 – α) % confidence interval for population correlation coefficient. (4+6)

17. A) Explain different types of errors involved testing of hypothesis with example. Deduce the relation between Type II Error and Power of the test.

    B) Show that maximum likelihood estimator and method of moment estimator of p, when X ~ Geometric (p) are equal. (5+5)

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