



Register number:

Date and session:

ST. JOSEPH'S UNIVERSITY, BENGALURU -27
B.Sc (MATHEMATICS) - III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November/ December 2023)
MT 322- MATHEMATICS III

(For current batch students only)

Time: 2 Hours

Max Marks: 60

This paper contains **TWO** printed pages and **THREE** parts.

PART A

Answer any **SIX** of the following.

[6X 2=12]

1. Let $G = \mathbb{Z}_6$ and $H = \{0, 2, 4\}$. Find all the distinct cosets of H in G .
2. Test the convergence of the sequence $\left\{ \frac{(3n+1)(n+2)}{n(n-1)} \right\}$.
3. Prove that the sequence $\left\{ \frac{3n+4}{2n+1} \right\}$ is monotonically decreasing.
4. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$.
5. Find the particular integral of the differential equation $(D^2 + 16)y = 14\cos 3x$.
6. Determine if the given differential equation $x^2y'' + 4xy' + 2y = e^x$ is exact or not.
7. Find the Laplace transform of $(e^{-3t}\cos 5t)$.
8. Find the inverse Laplace transform of $\left(\frac{s^2 - 3s + 4}{s^3} \right)$.

PART B

Answer any **THREE** of the following.

[3X 6=18]

9. Let a be an element of a group G . If $|a|$ is finite and equal to n , then prove that $\langle a \rangle = \{e, a, a^2, \dots, a^{n-1}\}$ and $a^i = a^j$, if and only if $n|(i-j)$.

10. Prove that every subgroup of a cyclic group is cyclic.
11. Prove that the limit of a convergent sequence is unique.
12. Show that the sequence $\{S_n\}$ defined by $S_1 = \sqrt{6}$ and $S_{n+1} = \sqrt{6S_n}$ converges to 6.

PART C

Answer any FIVE of the following.

[5X 6=30]

13. Solve the differential equation $x^3 \frac{d^3y}{dx^3} - 3x \frac{dy}{dx} + 3y = 4x$.
14. Solve the differential equation $\frac{d^2y}{dx^2} + \left(\frac{1}{x} - 2\right) \frac{dy}{dx} + \left(1 - \frac{1}{x}\right) y = 0$ when a part of the complementary function is given.
15. Solve the differential equation $y'' - 3y' + 2y = e^{-x}$ using the method of variation of parameters.
16. (a) Prove that the center $\mathbb{Z}(G)$ of the group G is a normal subgroup of G .
 (b) Evaluate $\int_0^{\infty} e^{-3t}(t \sin t) dt$. **[3+3]**
17. Find the inverse Laplace transform of the function $\left(\frac{s}{s^2 + s - 2}\right)$.
18. Verify convolution theorem for $f(t) = \sin t$ and $g(t) = e^{-t}$.
19. Using Laplace transform method, solve $9y'' - 6y' + y = 0$ given that $y(0) = 3$ and $y'(0) = 1$.

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