Register Number:

Date:

ST JOSEPH'S UNIVERSITY, BENGALURU-27
B.Sc (MATHEMATICS) - III Semester
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November/December 2023)
MTOE 8 -MATHEMATICS FOR PHYSICAL SCIENCES-III
(For current batch students only)

Duration: 2 Hours **Max. Marks:** 60

This paper contains TWO pages and THREE parts.

PART A

Answer any SIX of the following. **(6×2=12)**

1. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$.
2. Solve $\frac{d^2y}{dx^2} + 4y = 0$.
3. Check if the given differential equation is exact, $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$.
4. Find the reciprocal of $z = 2 - 3i$.
5. Define an analytic function and an entire function.
6. Find the real and imaginary parts of e^{x+iy}.
7. Find the gradient of $log(x^2 + y^2 + z^2)$.
8. Prove that the vector point function $\vec{F}(x, y, z) = z\ \hat{i} + x\ \hat{j} + y\ \hat{k}$ is solenoidal.

PART B

Answer any THREE of the following. **(3×6=18)**

9. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 5\cos 3x$.
10. Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = (x + 1)e^x$.
11. Solve the simultaneous differential equations, $\frac{dx}{dt} + 2y = \sin 2t$; $\frac{dy}{dt} - 2x = \cos 2t$.
12. Solve the Cauchy Euler differential equation $x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 13y = x^2$.
13. Solve by method of variation of parameters, $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{2x}$.

PART C

Answer any <u>FIVE</u> of the following. **(5×6=30)**

14. Check if the following functions are analytic.

    (i) $2xy + i(y^2 - x^2)$

    (ii) $2x^2y + i(y^2 - x)$ **(3+3)**

15. Check if the C-R equations are satisfied for the function $(r + \frac{1}{r})\cos\theta + i(r - \frac{1}{r})\sin\theta$.

16. Show that the real and imaginary parts of the function $f(z) = z^3$ are harmonic.

17. Find the analytic function $f(z) = u+iv$ using Milne Thomson method, where $u = x^3-3xy^2+3x^2-3y^2+1$.

18. Find the gradient of the scalar point function $\phi(x, y, z) = 3x^2 + 2y^3 - 5z$. Also find the directional derivative of ϕ at $(1, 2, -1)$ in the direction of $\hat{i} - \hat{j} + 2\hat{k}$.

19. (i) If $\vec{F} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + \lambda z)\hat{k}$ is solenoidal, find the value of λ.

    (ii) Prove that the vector point function $\vec{F}(x, y, z) = (6xy + z^3)\ \hat{i} + (3x^2 - z)\ \hat{j} + (3xz^2 - y)\ \hat{k}$ is irrotational. **(3+3)**

20. Find the divergence and curl of $\vec{F}(x, y, z) = 2x^2y\ \hat{i} + 3x\ \hat{j} + 4zx\ \hat{k}$.