Register Number:

Date:

ST JOSEPH'S UNIVERSITY, BENGALURU-27
M.Sc (MATHEMATICS) - I Semester
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November/December 2023)
MT 7521- DISCRETE MATHEMATICS AND GRAPH THEORY
(For current batch students only)

Duration: 2 Hours **Max. Marks:** 50

1. The paper contains two pages.
2. Attempt any **FIVE FULL** questions.

1. a) Suppose that either a member of Mathematics faculty or a student who is Mathematics major is chosen as a representative. If there are 37 members of Mathematics faculty and 83 students of Mathematics major and no one is both faculty and student then how many ways of choosing a representative to the University? **[2m]**

   b) How many different license plates can be made if each plate contains a sequence of three letters followed by four digits in the cases (i) with repetition of letters and digits. (ii) without repetition of letters and digits. **[2m]**

   c) Find the number of positive integers not exceeding 100 that are either odd or a perfect square of an integer. **[3m]**

   d) A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, 114 in Russian. Further, 103 have taken courses in both Spanish and French, 14 have taken courses in both French and Russian and 23 have taken courses in both Spanish and Russian. If 2092 students have taken at least one of Spanish, French, Russian, then how many have taken a course in all three languages? **[3m]**

2. a) Find the solution of a given recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$. **[5m]**

   b) Define an equivalence relation. Show that the relation congruent modulo n is an equivalence relation. **[5m]**

3. a) Give the Cartesian product of cycle C_3 and path P_3. **[2m]**

   b) Prove that a nontrivial connected graph G is Eulerian if and only if every vertex of G has even degree. **[8m]**

4. a) Define center of a tree. Prove that every tree of order n has size $n - 1$. **[5m]**

b) Define matrix tree theorem. Find the number of spanning trees of K_4 using matrix tree theorem and also give all its spanning trees. **[5m]**

OR

a) Explain Kruskal's algorithm. Find the minimum spanning tree of the graph in figure 1 using Kruskal's algorithm. **[4m]**

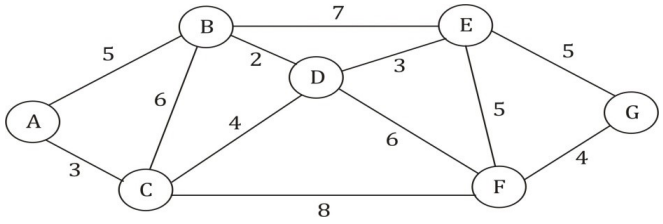

Figure 1: Weighted graph

b) Define vertex connectivity $\kappa(G)$ and edge connectivity $\lambda(G)$ of a graph G. Prove that for any graph, $\kappa(G) \leq \lambda(G) \leq \delta(G)$. **[6m]**

5. a) Define adjacency and incidence matrix of a graph. Give the adjacency and incidence matrix of the bipartite graph $K_{2,3}$. **[4m]**

   b) Define the vertex covering number $\alpha_0(G)$ and the vertex independence number $\beta_0(G)$ of a graph G. For any nontrivial connected graph G, prove that $\alpha_0(G) + \beta_0(G) = n$. **[6m]**

6. State and prove Five colour theorem. **[10m]**

7. (a) Define the chromatic polynomial of a graph. Find the chromatic polynomial of the graph given in figure 2. **[5m]**

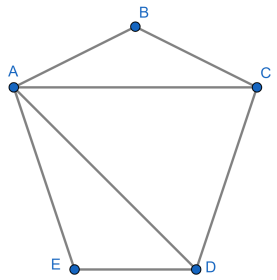

Figure 2:

(b) If G is a graph of order n, then prove that $\frac{n}{1+\Delta(G)} \leq \gamma(G) \leq n - \Delta(G)$ where $\gamma(G)$ is the domination number of G and $\Delta(G)$ is the maximum degree in G. **[5m]**