## ST JOSEPH'S UNIVERSITY, BENGALURU -27 <br> MSc PHYSICS - I SEMESTER <br> SEMESTER EXAMINATION: OCTOBER 2023 <br> (Examination conducted in November/December 2023) PH 7321 - NUMERICAL TECHNIQUES <br> (For current batch students only)

Time: 2 Hours
Max Marks: 50
This paper contains 2 printed pages and 2 parts

## PART-A

Answer any five questions. Each question carries 7 marks.

1. a. Find the cubic polynomial which takes the following value.

| $x$ | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| $f(x)$ | 1 | 2 | 1 | 10 |

Hence or otherwise evaluate $f(4)$
b. The derivative of a function $f(x)$ at a particular value of $x$ can be approximately calculated by

$$
\begin{equation*}
f^{\prime}(x) \approx \frac{f(x+h)-f(x)}{h} \tag{4+3}
\end{equation*}
$$

For $f(x)=7 e^{0.5 x}$ and $\mathrm{h}=0.3$, find the relative true error at $\mathrm{x}=2$.
2. Find the maximum and minimum value of $y$ from the following data.

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 2 | -0.25 | 0 | -0.25 | 2 | 15.75 | 56 |

3. The open loop response, that is, the speed of the motor to a voltage input of 20 V , assuming a system without damping is

$$
20=(0.02) \frac{d w}{d t}+(0.06) w
$$

If the initial speed is zero $(w(0)=0)$, and using Euler's method, what is the speed at $t=0.8 \mathrm{~s}$ ? Assume a step size of $h=0.4 \mathrm{~s}$. find also the exact solution and the relative true error.
4. a. Find the Fourier transform of the one-sided exponential function.

$$
f(t)=\left\{\begin{aligned}
0, & t \leq 0 \\
e^{-\alpha t}, & t>0
\end{aligned}\right.
$$

Where $\alpha$ is a positive constant. Also, sketch the function in the time domain. b. If $u(t)$ is used to denote the Heaviside unit step function:

$$
u(t)= \begin{cases}0, & t \leq 0 \\ 1, & t>0\end{cases}
$$

Then we can write the function $f(t)$ given in question 4a. as $f(t)=e^{-\alpha t} u(t)$.
Write down the Fourier transform of
(i) $e^{-t} u(t)$
(ii) $e^{-3 t} u(t)$
(iii) $e^{\frac{t}{2}} u(t)$
5. Calculate the coefficient of correlation for the ages of husbands and their respective wives using the assumed mean method with the data given below. Also, interpret your result.

| Age of <br> husbands | 23 | 27 | 28 | 29 | 39 | 31 | 33 | 35 | 36 | 39 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Age of <br> wives | 18 | 22 | 23 | 24 | 25 | 26 | 28 | 29 | 30 | 32 |

6. A factory has two machines A and B. Records show that machine A produced $60 \%$ of the items of output and machine B produced $40 \%$ of the items. Further, $2 \%$ of the items produced by machine A were defective and $1 \%$ produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. Use Bayes Theorem to find the probability that it was produced by machine B.
7. A random variable $X$ has to follow the probability function as shown below

| $X$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $P(X)$ | 0 | $k$ | $2 k$ | $2 k$ | $3 k$ | $k^{2}$ | $2 k^{2}$ | $7 k^{2}+k$ |

(i) Find the value of the k
(ii) Evaluate $P(X<6), P(X \geq 6)$
(iii) $\quad P(0<X<5)$

## PART B

Answer any three questions. Each question carries 5 marks.
8. Show that $e^{\frac{x^{2}}{2}}$ is self-reciprocal under the Fourier cosine transform.
9. Evaluate $\int_{0}^{3}\left(2 x-x^{2}\right) d x$, taking 6 intervals by (i) Trapezoidal rule and (ii) Simpson's $\frac{1}{3}$ rule. Also, calculate the true value and relative true error.
10. A dice is tossed thrice. In each throw, getting 1 or 6 would be considered as success. Find the mean and variance of the number of successes.
11. (a) An incomplete frequency distribution is given below:

| Variable | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 12 | 30 | $?$ | 65 | $?$ | 25 | 18 |

Given that the total frequency is 229 and median is 46 , find the missing frequencies.
(b) A problem in mechanics is given to three students $\mathrm{A}, \mathrm{B}$ and C whose chances of solving it are $1 / 2,1 / 3$ and $1 / 4$ respectively. What is the probability that the problem will be solved.

PH 7321_B_23

