Registration Number:

Date & session:

ST JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc. (STATISTICS) – I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November /December 2023)

ST 7421: MATHEMATICAL ANALYSIS AND LINEAR ALGEBRA
(For current batch students only)

Time: 2 Hours **Max Marks: 50**

This paper contains TWO printed page and ONE part.

PART-A

I. Answer any FIVE questions out of SEVEN questions:

1. A) Define open set. Prove that union of any collection of open sets is open.
   B) Prove that limit of a sequence is unique if it exists. Given an example of sequence of real numbers which is bounded but not convergent. (5+5)

2. A) Prove Weierstrass M-test for the Uniform convergence of series.
   B) Verify whether $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converges or not.
   C) Define Riemann-Stieltjes (R-S) integral. (4+3+3)

3. A) If $f \in R(\alpha)$ and $g \in R(\alpha)$ then prove that $c_1 f + c_2 g \in R(\alpha)$ where c_1 and c_2 are real numbers.
   B) Discuss the convergence of (i) $\int_0^2 \frac{dx}{(2x-x^2)}$ (ii) $\int_0^1 \frac{dx}{(1-x)}$ (6+4)

4. A) State and prove sufficient condition for $f \in R(\alpha)$ on [a, b]. .
   B) State mean value theorems for R-S integral.
   C) State Legendre's duplication formula (5+3+2)

5. A) With usual notation prove that $B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$.
   B) Briefly explain Lagrangian multiplier technique (6+4)

6. A) Evaluate the double integral $\int_0^2 \int_0^1 (1 - x^3) \mathrm{d}x \, \mathrm{d}y$

   B) Define Vector space and write all the ten properties of vector space.

   C) Find the inner product of the vectors X_1=(1, 1, 0) and X_2 = (1, -1, 0) and comment.

(2+5+3)

7. A) Explain Gram-Schmidt Orthogonal Process.

   B) Find the inverse of the matrix. $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & -4 \end{bmatrix}$.

   C) Define Idempotent matrix and show that the following matrix is Idempotent.

$$B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

(2+5+3)

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