



Register Number:

Date and session:

**ST JOSEPH'S UNIVERSITY, BENGALURU - 27**  
**M.Sc (MATHEMATICS) - 2<sup>nd</sup> SEMESTER**  
**SEMESTER EXAMINATION: APRIL 2024**  
(Examination conducted in May/June 2024)  
**MT 8321 - COMPLEX ANALYSIS**  
(For current batch students only)

**Duration:** 2 Hours

**Max Marks:** 50

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1. The paper contains **TWO** printed pages and **ONE** part.
  2. Answer any **FIVE FULL** questions.
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1. a) State and prove Cauchy's integral formula.  
b) Find the maximum modulus of  $f(z) = 2z + 5i$  on the closed circular region defined by  $|z| \leq 2$ . [6+4]
2. a) State and prove Taylor's theorem.  
b) Prove Maximum modulus theorem. [6+4]
3. a) Expand  $f(z) = \frac{5}{z^2 + z - 6}$  in a Laurent's series valid for  $2 < |z| < 3$ . [5+5]  
b) Find and classify the singularities of the functions  
(i).  $f(z) = \frac{1 - \cos z}{z^3}$  at  $z = 0$ . (ii).  $f(z) = \frac{z}{1 - \sin z}$  at  $z = 0$ .

**OR**

- a) Use Residue theorem to evaluate  $\int_C \frac{(z^2 + 1)dz}{(z - 1)(z - 2)^2(z + 5)}$  where  $C$  is the circle with  $|z| = 4$ .
- b) Use Residue theorem to evaluate  $\int_0^{2\pi} \frac{d\theta}{(2 + \cos\theta)}$ .

[5+5]

4. a) Evaluate  $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)(x^2 + 4)}$ .
- b) Evaluate  $\int_0^{\infty} \frac{\cos x dx}{(x^2 + 1)}$ . [5+5]
5. a) Define holomorphic function with an example.
- b) Suppose  $f$  is analytic at  $z_0$  and  $f'(z_0) \neq 0$ , prove that  $f$  is conformal and locally  
1-1 at  $z_0$ . [2+8]
6. a) Define Zero of an analytic function.
- b) State and prove Rouché's theorem. [2+8]
7. a) If  $S$  is a circle or line, and  $f(\frac{1}{z})$  then prove that  $f(S)$  is also a circle or a line.
- b) Define Meromorphic function with an example. [7+3]

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