Register number:

Date and session:

ST JOSEPH'S UNIVERSITY, BENGALURU-27
M.Sc MATHEMATICS - 4^{th} SEMESTER
SEMESTER EXAMINATION: APRIL 2024
(Examination conducted in May/June 2024)
MT 0122- ADVANCED GRAPH THEORY
(For current batch students only)

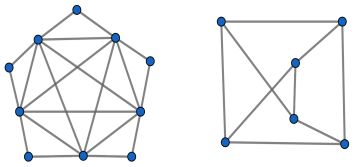
Duration: 2 Hours **Max. Marks:** 50

1. The paper contains **TWO** pages.
2. Attempt any **FIVE FULL** questions.

1. (a) Define the diameter and the periphery of a graph. Find the periphery of the Petersen graph. **[5m]**

   (b) Prove that a connected graph G of order $n \geq 2$ has locating number $(n-1)$ if and only if G is isomorphic to the complete graph K_n. **[5m]**

2. (a) Check if the following graphs are planar. Justify your answer. **[5m]**

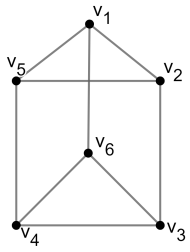


   (b) Prove that if G is maximal outer planar graph with order $n \geq 3$, then G has $n-2$ interior regions. **[5m]**

3. (a) Prove that a graph has a dual if and only if it is planar. **[10m]**

OR

   (b) Define vertex transitive graphs. Is the graph given below vertex transitive? Justify. **[5m]**



(c) Define circulant graphs. Give a circulant graph isomorphic to complete graph K_8 and cycle C_8. **[5m]**

4. Let G be a connected vertex-transitive graph with minimum degree $\delta(G)$ and edge connectivity $\lambda(G)$, then prove that $\lambda(G) = \delta(G)$. **[10m]**

5. (a) Prove that any De Bruijn digraph $B(d, n)$ contains Euler circuits and Hamilton cycles. Also, find an Euler circuit in $B(2, 3)$. **[5m]**

   (b) Define Butterfly networks $BF(n)$. Draw $BF(2)$, $BF(3)$ and $BF(4)$. **[5m]**

6. Prove that the direct product and the strong product are each associative. **[10m]**

7. Prove the distance formula for the lexicographic product of graphs. **[10m]**