ST.JOSEPH'S UNIVERSITY, BENGALURU -27
B.Sc. (MATHEMATICS) – II SEMESTER
SEMESTER EXAMINATION: APRIL 2024
(Examination conducted in May/June 2024)
MT 221 – MATHEMATICS- II
(For current students only)

Reg. Number:
Date & session:

Time: 2 Hours **Max Marks: 60**

This paper contains 1 printed page and 3 parts

PART A

Answer any Six of the following: **(6X2=12)**

1. Show that the inverse of an element in a group is unique.
2. On the set of positive rational numbers Q^+, the binary operation $*$ is defined by $a * b = \frac{ab}{2}$. Find the identity element and inverse of 4.
3. Find the area bounded between the cissoid $y^2(a-x) = x^3, a > 0$ and its asymptote.
4. Find the slope of the tangent to the curve $r = a\, sin2\theta$ at the point $\theta = \frac{\pi}{4}$.
5. Show that the curves $r = ae^{\theta}$ and $re^{\theta} = b$ intersect orthogonally.
6. Find the envelopes of the family of circles, whose centre lies on the x-axis.
7. Solve $x\frac{dy}{dx} - 2y = 2x$.
8. Find the singular solution of $y = px + \frac{a}{p}$.

PART B

Answer any three of the following: **(3X6=18)**

9. Show that $G = \{2,4,6,8\}$ forms an abelian group under $\times_{10}$ by using Cayley's table.
10. If a is a generator of a cyclic group G then show that $O(a) = O(G)$.
11. Evaluate (i) $\int_0^{2a} x^2\sqrt{2ax - x^2}\, dx$ (ii) $\int_0^{\infty} \frac{x^4}{(1+x^2)^4} dx$. (4+2)
12. Find the surface area of the solid obtained by revolving the cardioid $r = a(1 + cos\theta)$ about the initial line.

PART C

Answer any five of the following: **(5X6=30)**

13. Derive the formula for the derivative of arc length for the cartesian equations.
14. Show that the pedal equation of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $r^2 = a^2 - 3p^2$.
15. Find all the asymptotes of the curve $2x^3 - x^2y - 2xy^2 - 4x^2 + 8xy - 4x + 1 = 0$.
16. (a) Derive the formula for the radius of curvature of cartesian curves.

    (b) Solve $\left[y\left(1 + \frac{1}{x}\right) + cosy\right]dx + [x + logx - xsiny]dy = 0$. (3+3)
17. Find the suitable integrating factor and solve $y(8x - 9y)dx + 2x(x - 3y)dy = 0$.
18. Reduce the equation $(x^2 - 1)p^2 - 2xyp + y^2 - 1 = 0$ into Clairaut's form and find the general solution.
19. Show that the family of parabolas $y^2 = 4a(x + a)$ is self-orthogonal.
