



Register Number:

Date:

ST JOSEPH'S UNIVERSITY, BENGALURU-27
B.Sc OPEN ELECTIVE (MATHEMATICS) - 4th Semester
SEMESTER EXAMINATION: APRIL 2024
(Examination conducted in May/June 2024)
MTOE 11 -MATHEMATICS FOR PHYSICAL SCIENCES-IV
(For current batch students only.)

Duration: 2 Hours

Max. Marks: 60

This paper contains TWO pages and THREE parts.

PART A

Answer any **SIX** of the following.

(6×2=12)

1. Check the integrability condition for the total differential equation $xdx + ydy + zdz = 0$.
2. Form the partial differential equation by eliminating arbitrary constants a and b from the equation $z = ax + by + ab$.
3. Check if the function $x^5 \cos^3(x)$ is even or odd in $(-\pi, \pi)$.
4. Prove that $\Gamma(1) = 1$.
5. Prove that Beta function is symmetric.
6. State Euler-Lagrange equation for variational problems.
7. Write the name and equation of the path on which a particle in the absence of friction will slide from one fixed point to another fixed point in the shortest time under the action of gravity.
8. Define geodesic and write the name of the geodesic on a right circular cylinder.

PART B

Answer any **THREE** of the following.

(3×6=18)

9. Check for integrability and solve $xdx + y^2dy + \frac{dz}{z}$.
10. Solve the simultaneous differential equation $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$.
11. Obtain the Fourier series of the function $f(x) = x$ in the interval $(-\pi, \pi)$.
12. Obtain the general solution of the extremal problem $I(y) = \int_a^b [xy' + (y')^2] dx$.
13. Find the shortest smooth plane curve joining any two distinct points $P(x_1, y_1)$ and $Q(x_2, y_2)$.

PART C

Answer any FIVE of the following.

(5×6=30)

14. Solve $p + q = \sin(x)$.
15. Solve $(D^2 - 2DD' + D'^2)z = e^{x+2y}$.
16. Obtain the Fourier half range sine series for the function f defined by $f(x) = (x - 1)^2$ in $(0, 1)$.
17. Evaluate the following integrals.

i) $\int_0^{\infty} e^{-x^2} x^{2n-1} dx, n > 1$

ii) $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^{\frac{5}{2}} x dx$

18. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, where $m, n > 0$.
 19. Find the equation of the curve formed by a heavy chain when it is suspended under the action of gravity between two fixed points.
 20. Find the extremal of the functional $I(y) = \int_0^{\pi} [(y')^2 - y^2] dx$, under the boundary condition $y(0) = 0, y(\pi) = 1$ subjected to the constraint $\int_0^{\pi} y dx = 1$.
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