Register Number:

Date and Session:

ST. JOSEPH'S COLLEGE(AUTONOMOUS), BENGALURU -27
B.Sc (MATHEMATICS) - VI SEMESTER
SEMESTER EXAMINATION: APRIL 2024
(Examination conducted in May/June 2024)

MT 6123 - MATHEMATICS VII

(For current batch students only)

Time: 2 Hours　　　　Max Marks: 60

This paper contains **TWO** printed pages and **THREE** parts.

PART A

Answer any SIX of the following. **[6 x 2=12]**

1. If $H = \{(x, 0)|x \in \mathbb{R}\}$ and $K = \{(0, y)|y \in \mathbb{R}\}$ are any two subspaces of $\mathbb{R}^2$. Justify why $H \cup K$ is not a subspace.
2. Is the set $S = \{(1, 3, -1), (3, 9, -3)\}$ of $\mathbb{R}^3$ a linearly dependent set over $\mathbb{R}$? Justify your answer.
3. Write the standard basis and dimension of $P_4(F)$, which is a vector space of polynomials whose degree is less than or equal to 4.
4. Show that $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ defined by $T(x, y) = (x^2, y^2)$ is not a linear transformation.
5. Define range space and null space of a linear transformation.
6. By eliminating the arbitrary constants, form the partial differential equation from the relation $z = (x^2 + a)(y^2 + b)$.
7. Find the complete integral of $z = px + qy + \sqrt{pq}$.
8. Solve: $(D^2 - 7DD' + 6D'^2)z = 0$.

PART B

Answer any FIVE of the following. **[5 x 6=30]**

9. Let V be a vector space over a field F and W be a subset of V. Prove that W is a subspace of V if and only if the following three conditions hold for the operations defined in V.
   a) $0 \in W$
   b) $x + y \in W$ whenever $x \in W$ and $y \in W$
   c) $cx \in W$ whenever $c \in F$ and $x \in W$
10. Show that a subset W of a vector space V is a subspace of V if and only if $span(W) = W$.
11. Determine whether $\{x^3 - x, 2x^2 + 4, -2x^3 + 3x^2 + 2x + 6\}$ in $\mathbb{P}_3(\mathbb{R})$ is linearly dependent or linearly independent.
12. Let V be a vector space over a field F with dimension n. Prove the following statements.
   a) Any linearly independent subset of V that contains exactly n vectors is a basis for V.
   b) Every linearly independent subset of V can be extended to a basis for V.

13. Let U and V be vector spaces over the same field F with $dimension(U) = p$. If $T : U \to V$ is linear, then prove that $rank(T) + nullity(T) = p$.

14. Find the linear transformation for the matrix $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}$ with respect to the bases $B_1 = \{(1, -1), (1, 1)\}$ and $B_2 = \{(1, 0), (0, 1)\}$.

15. Let T be the linear operator on $\mathbb{P}_1(\mathbb{R})$ defined by $T(p(x)) = p'(x)$, the derivative of $p(x)$.
Let $B_1 = \{1, x\}$ and $B_2 = \{1 + x, 1 - x\}$.

    a) Find the change of coordinate matrix Q which changes B_2 coordinates into B_1 coordinates and Q^{-1}.
    b) Compute $[T]_{B_1}$
    c) Find $[T]_{B_2}$ using Q and $[T]_{B_1}$

PART C

Answer any THREE of the following. **[3 x 6=18]**

16. Check the integrability and solve: $zdx + zdy + (x + y + sinz)dz = 0$.

17. Form the partial differential equation by eliminating arbitrary function:$\phi(x^2 + y^2 + z^2, ax + by + cz) = 0$.

18. Solve the partial differential equation: $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$.

19. Solve the partial differential equation by Charpit's method: $p^2x + q^2y = z$.

20. Solve:$(D^2 - DD')z = x^3y$.

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