Registration No.:
Date and Session::

ST.JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc. (PHYSICS) – II SEMESTER
SEMESTER EXAMINATION: APRIL 2024
(Examination Conducted in May 2024)

PH 8421-Quantum Mechanics-I

Time: 2 **hours** **Maximum marks: 50**

This question paper has 2 printed pages and 2 parts

PART A

Answer any **FIVE** of the following questions. Each question carries 7 marks. $[5 \times 7 = 35]$

1. (a) Find the ground state energy level of an electron confined to a one-dimensional infinite potential well of length L = 0.1 nm. [4]

   (b) Make the energy level diagram and find the wavelength of the photons emitted for all the transitions beginning at n=3 state or less and ending at a lower energy level. [3]

2. From the Schrodinger's equation for the Hydrogen atom, obtain the solution for the azimuthal part and demonstrate that the solution is associated with the magnetic quantum number. [7]

3. (a) Show that $R_{10}(r) = \frac{2}{a_0^{(3/2)}} e^{\frac{-r}{2a_0}}$ is a solution for the radial part of the differential equation of the hydrogen atom given below:

   $\frac{1}{r^2}\frac{d}{dr}(r^2\frac{dR}{dr}) + [\frac{2m}{\hbar^2}(\frac{e^2}{4\pi\epsilon_0 r} + E) - \frac{l(l+1)}{r^2}]R = 0.$ [5]

   (b) Show that $R_{10}(r)$ is normalized. [2]

4. (a) Derive the equation for quantum harmonic oscillator from the time independent Schrodinger's equation. [4]

   (b) Prove that the time evolution of a state from $\mathtt{t} = 0$ to T is given by an unitary operator U. [3]

5. (a) Show that if an operator commutes with the Hamiltonian, then the expectation value of that operator is conserved. [4]

   (b) Find the expectation value of $[x, p_x]$. [3]

6. For a 1-D quantum harmonic oscillator, prove that $b^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$ and $b|n\rangle = \sqrt{n}\,|n-1\rangle$, where b, $b^\dagger$ are annihilation and creation operators respectively.
   Given:- The associated Hamiltonian in terms of b and $b^\dagger$ is $H = \hbar\omega(b^\dagger b + 1/2)$ and it satisfies the condition $H|n\rangle = E_n|n\rangle$ [7]

7. (a) If J_x, J_y, J_z are generalised operators, what condition should be satisfied by them so that they will be termed as *Anugular Momentum* operators? [2]

   (b) If $J_+ = J_x + iJ_y$ and $J_- = J_x - iJ_y$ are the raising and lowering operators respectively, prove that

      i. $[J_z, J_+] = \hbar J_+$
      ii. $[J^2, J_+] = 0$

   [5]

PART B

Answer any **THREE** of the following questions. Each question carries 5 marks. $[3 \times 5 = 15]$

8. Use the Uncertainty Principle to estimate the zero point energy of a quantum harmonic oscillator.

9. An electron in a 2-D infinite square well potential needs to absorb electromagnetic wave with a wavelength 4040 nm to be excited from its first excited state to next higher energy level. Find the length of the box.

10. If $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, find $[\sigma_x, \sigma_y]$.

11. Find $< J_+ >$ in the state $\frac{1}{\sqrt{3}}(|1,1\rangle + |1,0\rangle + |1,-1\rangle)$.
    Given:- $J_+ |j,m\rangle = \hbar\sqrt{j(j+1) - m(m+1)}\, |j,m+1\rangle$