Register Number: DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - III SEMESTER

SEMESTER EXAMINATION- OCTOBER 2019

For supplementary candidates only.

PH 9115 - QUANTUM MECHANICS II

Time-2 1/2 hrs. **Maximum Marks-70**

This question paper has 4 printed pages and 2 parts and a table of constants and integrals

PART A

Answer any FIVE full questions. **(5x10=50)**

1. Consider a system made up of two identical particles a and b each located at positions $\vec{r}_1$ and $\vec{r}_2$ respectively and having wavefunctions: $\psi_a(\vec{r}_1)$ and $\psi_b(\vec{r}_2)$ respectively. Develop the composite wavefunction for the system if the particles are indistinguishable bosons.

2.
   (a) Using the Variational Principle, obtain the estimate of the ground state energy of a particle in a one dimensional box. Assume the wavefunction to have a form of: $\psi_t = A e^{-bx^2}$

   (**5 Marks**)

   (b) The WKB approximation with the connection formulae result in the following expression for the wave function: $\psi(x) = \begin{cases} \frac{D}{\sqrt{p(x)}} \sin \frac{1}{\hbar}\left(\int_x^{x_2} p(x')\,dx' + \frac{\pi}{4}\right) & x < x_2 \\ \frac{D}{\sqrt{|p(x)|}} e^{-\frac{1}{\hbar}\left(\int_{x_2}^{x} |p(x')|\,dx'\right)} & x > x_2 \end{cases}$. Obtain the energy of the bound states (energy levels) of a particle in a box with a hard wall at $x=0$ and a sloping floor potential of the form $V(x) = \begin{cases} kx & \text{if } x>0 \\ \infty & \text{otherwise} \end{cases}$ where k is a positive constant.

   (**5 Marks**)

3. Explain perturbation theory and obtain the first order corrections to the energy for a system having its state space described by non-degenerate eigenstates.
4. Obtain the determinant (for first order correction to the energy) of a system having a three fold degenerate energy level.
5.
   (a) Starting with the time dependent Schrodinger Equation:
   $$i\hbar\frac{\partial}{\partial t}\Psi(\vec{\mathbf{r}},t)=H(\vec{\mathbf{r}},t)\Psi(\vec{\mathbf{r}},t) \quad ,$$
   using separation of variables $\Psi(\vec{\mathbf{r}},t)=\phi(\vec{\mathbf{r}})\,\psi(t)$, obtain the Schrodinger equation for time dependent perturbation of the type: $H(t)=H_0+\lambda\hat{W}(t)$.
   (b) If we assume that at time $t=0$, $|\psi(t=0)\rangle=|\phi_i\rangle$, and that for any time $t\neq 0$ we assume $|\psi(t)\rangle=\sum_n c_n(t)|\phi_n\rangle$, what will the Schrodinger equation reduce to?
6.
   (a) Explain scattering by a potential. What are the assumptions used in order to develop the scattering theory? (**4 Marks**)
   (b) What is scattering cross section? Write down the expression for differential cross section and work out the asymptotic form for the scattered wave. (**6 Marks**)
7. Derive the Hamiltonian of a system of two particles in a potential (for the scattering problem) dependent on the distance between the two in the center of mass frame. Express the Hamiltonian in reduced mass, and explain all terms.

PART B

Answer any <u>FOUR</u> full questions. (**<u>4x5=20</u>**)

8. Two noninteracting particles of mass m are in a one dimensional infinite potential well. Remember, the wavefunction and energies for one particle in an infinite potential are given as: $\psi_n(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$ and $E_n=\frac{\hbar^2}{2m}\frac{n^2\pi^2}{L^2}$. What are the wavefunction and energies of the two particles if:
   a) both are in the ground state? What are these for Fermions and Bosons respectively? (**2 Marks**)
   b) one is in the ground state and the other in the first excited state? What are these for Fermions and Bosons? (**3 Marks**)
9. Using the Variational Principle, estimate the ground state energy of a Simple Harmonic Oscillator (with potential $V(x)=\frac{1}{2}m\omega^2x^2$). Use a trial wavefunction of the form $\psi_t(x)=\frac{A}{x^2+b}$. You may take the expectation value of the Kinetic Energy to be given as: $\langle T\rangle=\frac{\hbar^2}{2m}\frac{1}{2b}$.
10. A particle is trapped in a well. External to this well, the potential has the form $V(x)=-kx$. Draw a figure, clearly demarcating the turning points. Obtain the expression for the probability of the particle tunneling out of the potential.
11. Calculate the first order correction to $E_3^{(0)}$ for a particle in a one-dimensional box with walls

at $x=0$ and $x=L$ due to a perturbation: $W=\lambda \hat{W}=10^{-3} E_1 \frac{x}{L}$.

12. We know that for the time dependent perturbation, the component of the state $|\phi_k\rangle$ to which the state $|\phi_i\rangle$ transitions to is given to a first order as: $b_k^{(1)}(t)=\frac{1}{i\hbar}\int_0^t e^{i\omega_{ki}t'}\langle\phi_k|\hat{W}|\phi_i\rangle dt'$. Furthermore, the transition probability from an initial state $|\phi_i\rangle$ to a final state $|\phi_f\rangle$ can be deduced from this as $P_{if}=\lambda^2|b_f^{(1)}(t)|^2$. For $\hat{W}(t)=\hat{W}_0$ (a constant) compute P_{if} .

13. Use the Born approximation ($f_k^{(B)}(\theta,\phi)=\frac{2\mu}{\hbar^2 K}\int r' V(r')\sin(Kr')dr'$) to calculate the differential cross-section (remember, that the expression above is the scattering amplitude) for a central potential of the form $V(\vec{r})=\beta\frac{e^{-\alpha r}}{r}$ (which is the Yukawa potential).

(Some) Physical Constants

[Constants: **h**=6.626070x10^{-34} J s (**Planck's constant**), **1eV** = 1.6x10^{-19} J (**electron volt to Joules**), **c**=2.99792458x10^{8} m/s (**speed of light**), **1Å** = 1x10^{-10}m (**Angstrom to meters**), **e** = 1.602176x10^{-19} C (**electronic charge**), **ε₀**= 8.85418782x10^{-12}m^{-3}kg^{-1}s^{4}A^{2} (**permittivity of free space**), **m$_{proton}$**=1.672621898x10^{-27}kg (**mass of proton**), **m$_{electron}$**=9.10938356x10^{-31}kg (**mass of electron**), **m$_{neutron}$**=1.674927471x10^{-27}kg (**mass of neutron**), **a** = 5.029x10^{-10}m (**Bohr radius**), **α** = 1/137 (**Fine Structure Constant**), **G**=6.674x10^{-11}m^{3}kg^{-1}s^{-2} (**Gravitational constant**), **M$_\odot$**=1.9891x10^{30} kg (**Solar mass**), **R$_\odot$**=6.9x10^{8} m (**Sun's Radius**), **σ** = 5.67x10^{-8} W m^{-2} K^{-4} (**Stefan-Boltzmann constant**), **M$_{Earth}$**=5.97x10^{27}kg (**Mass of Earth**), **D$_{earth-sun}$**=1.49x10^{11}m (**Earth-Sun distance**), 1 inch = 2.54 cm, 1foot=12 inches]

(a) Table of (some) Integrals

Gamma Function:
$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$
$\Gamma(n)=(n-1)!$
$\Gamma(\frac{1}{2}+n)=\frac{(2n)!}{4^n n!}\sqrt{\pi}$

(a) $\int_0^\infty e^{-2bt}dt=\frac{1}{2b}$

(b) $\int_0^\infty t\,e^{-2bt}dt=\frac{1}{4b^2}$

(k) $\int\frac{t^2}{(t^2+b^2)^2}dt=\left(-\frac{t}{(2b^2+2t^2)}+\frac{1}{2b}\tan^{-1}\left(\frac{t}{b}\right)\right)$

(l) $\int\frac{1}{(t^2+b^2)^3}dt=\frac{3}{8b^5}\left(\frac{5/3b^3t+bt^3}{(b^2+t^2)^2}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(c) $\int_0^\infty t^2 e^{-2bt}\,dt = \frac{1}{4b^3}$

(d) $\int_0^\infty t^3 e^{-2bt}\,dt = \frac{3}{8b^4}$

(e) $\int_0^\infty t^4 e^{-2bt}\,dt = \frac{3}{4b^5}$

(f) $\int_0^\infty t^5 e^{-2bt}\,dt = \frac{15}{8b^6}$

(g) $\int_0^\infty t^6 e^{-2bt}\,dt = \frac{45}{8b^7}$

(h) $\int \frac{1}{t^2+b^2}\,dt = \frac{1}{b}\tan^{-1}\left(\frac{t}{b}\right)$

(i) $\int \frac{1}{(t^2+b^2)^2}\,dt = \frac{1}{2b^3}\left(\frac{bt}{(b^2+t^2)} + \tan^{-1}\left(\frac{t}{b}\right)\right)$

(j) $\int \frac{1}{(t^2+b^2)^4}\,dt = \frac{1}{16b^7}\left(\frac{15t^5 b + 40b^3 5^3 + 33b^5 t}{(3t^6+9bt^4+9b^3t^2+b^5)} + 5\tan^{-1}\left(\frac{t}{b}\right)\right)$

(m) $\int \frac{t^2}{(t^2+b^2)^4}\,dt = \frac{1}{16b^5}\left(\frac{bt^5 + 8/3\, b^3 t^3 - 3b^5 t}{(b^2+t^2)^3} + \tan^{-1}\left(\frac{t}{b}\right)\right)$

(n) $\int \sqrt{a/x-1}\,dx = x\sqrt{a/x-1} + a\ \tan^{-1}\left(\sqrt{a/x-1}\right)$

(o) $\int \sqrt{1-ax}\,dx = -\frac{2(1-ax)^{3/2}}{3a}$

(p) $\int \sqrt{1-ax^2}\,dx = \frac{1}{2}x\sqrt{1-ax^2} + \frac{\sin^{-1}\sqrt{a}\,x}{2\sqrt{a}}$

(q) $\int_{-\infty}^{\infty} e^{-\alpha t^2 + i\omega t}\,dt = \sqrt{\frac{\pi}{\alpha}}\, e^{-\frac{\omega^2}{4\alpha}}$

(r) $\int_0^\infty t^4 e^{-\alpha^2 t^2}\,dt = \frac{3\sqrt{\pi}}{8\alpha^5}$

(s) $\int_0^\infty t^n e^{-st}\,dt = \frac{n!}{s^{n+1}}$ (Laplace Transform)