# ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27 M.Sc. PHYSICS - III SEMESTER SEMESTER EXAMINATION: OCTOBER 2019 <br> PH 9318: MODERN OPTICS 

Time: 2 1/2 hours
Max Marks:70
This paper contains 2 parts and 2 printed pages.

## Part-A

Answer any 5 questions. Each carries 10 marks.
$(5 \times 10=50)$

1. A light wave propagating in the z-direction is incident on a material medium in which each atom can be thought of as a classical forced oscillator being driven by time varying electric field applied in $x$-direction. Assuming the electric field to have frequency $\omega$, derive the dispersion equation for isotropic medium assuming the electrons to be damped harmonic oscillators.
2. Given that the convolution of two rectangular step functions is a triangular function and the Fourier transform of each rectangular step function a sinc function. Show that the Fourier transform of the triangular function which is represented as
$\mathrm{f}(\mathrm{x})=\left\{\begin{array}{lll}1-|x| / b & \text { when } & |x| \leq b \\ 0 & \text { when } & |x|>b\end{array}\right.$ is a $(\operatorname{sinc})^{2}$ function and hence justify convolution theorem.
3. In a single slit Fraunhofer diffraction pattern, the field distribution at a point on the screen is given as $E=A \frac{\sin \beta}{\beta} \cos (\omega t-\beta)$ where $\beta=\frac{\pi b \sin \theta}{\lambda}, b$ being the width of slit and
$\theta$ being the angle that each ray of diffracted light makes with the normal to the grating. Using this result, derive an expression for resultant intensity obtained from a double slit separated by distance ' $d$ ' between the slits. Explain the terms in the intensity relation and find the missing orders in the spectra if $d=2 b$.
4. a) Using Young's double hole experiment, explain the concept of spatial and temporal coherence.
b) Show that for an extended source S , if its linear dimension is $\frac{\lambda a}{d}$ (where $\lambda$ is the wavelength of the source, $a$ is the distance from the source to the pin holes and $d$ is the distance between the pin holes), the interference fringes formed in Young's double hole experiment for two equidistant pin holes $S_{1}$ and $S_{2}$ will not be seen.
5. a) Explain how can we experimentally differentiate between Linearly polarized, Circularly polarized and unpolarized light coming from a source.
b) Explain what is angular dispersion and resolving power of a grating. What is the difference between the two?
6. For a plane wave propagating at an angle $\psi$ to the optic axis(z-axis) in the $x-z$ plane in a uniaxial crystal, using this set of three homogenous equations:
$\left(\frac{k_{x}}{n^{2}}-\cos ^{2} \psi\right) E_{x}+\sin \psi \cos \psi E_{Z}=0$
$\left(\frac{k_{y}}{n^{2}}-1\right) E_{y}=0$
$\sin \psi \cos \psi E_{x}+\left(\frac{k_{z}}{n^{2}}-\sin ^{2} \psi\right) E_{z}=0$ where $k_{x}, k_{y}, k_{z}$ are the principal refractive indices along the three principal axis $x, y, z$ respectively, show that one of the solution to these equations correspond to the extra-ordinary wave propagating within the medium( Given :
$\vec{D}=\frac{1}{\omega}(\vec{H} \times \vec{k}) \quad \vec{H}=\frac{1}{\omega \mu_{o}}(\vec{k} \times \vec{E})$. For this wave, find the expression for the angle between the electric field vector $\vec{E}$ and the displacement vector $\vec{D}$ and show that the two vectors are not along the same direction. Also, interpret your result.
7. Using Maxwell's equations in dielectric medium, arrive at the inhomogeneous wave equation (in terms of $\vec{E}$ ) in a non-linear medium neglecting the anisotropy of the medium. Now, representing the real electric field as the sum of the complex field and its conjugate term, find the second order non-linear polarization at $\omega$ and $2 \omega$ and write the solution for wave equation at $2 \omega$ frequency.

## Part-B

Answer any 4 questions. Each carries 5 marks.
(5x4=20)
8. Two thin convex lenses of focal lengths 30 cm and 10 cm are separated by a distance of 10 cm . If an object of length 2 cm is placed at a distance of 20 cm to the left of the first lens, find the position of the image and the magnification.
9. In a spectrum obtained from plane transmission grating, find which spectral line in the fourth order will overlap with the third order line of 5461 $\AA$.
10. In the Young's double slit experiment if each slit is assumed to be a circular aperture of infinitesimal diameter separated by distance ' d ' with each slit located at $x= \pm d / 2$ then show that the Fourier transform of sum of these two functions from the two apertures is a cosine function which corresponds to the interference of light waves on the screen.
Given: Properties of delta function $\int_{-\infty}^{+\infty} \delta(x) f(x) d x=f(0)$ and $\int_{-\infty}^{+\infty} \delta\left(x-x_{o}\right) f(x) d x=f\left(x_{o}\right)$.
11. A 500 nm light wave in vacuum enters a glass plate of refractive index 1.60 and propagates perpendicularly across it. How many waves span the glass if its thickness is 1 cm ?
12. A Michelson's stellar interferometer is used to measure the angular dimension of a star. If the mirror separation is 6 m when the fringes disappear, calculate the angular diameter of a star measurable with a light of $\lambda=600 \mathrm{~nm}$.
13. A quartz plate of thickness $29.8 \mu \mathrm{~m}$ is cut parallel to the optic axis. The refractive index of quartz are $n_{0}=1.54617$ and $n_{e}=1.55535$ for the wavelength of $5461 \AA$. A linearly polarized light is incident on this plate with the E-vector making an angle of $45^{\circ}$ with the optic axis. Find the nature of polarization of light emerging from the plate.

