# ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27 <br> M.A. ECONOMICS- I SEMESTER <br> SEMESTER EXAMINATION: OCTOBER 2018 EC 7418: MATHEMATICAL METHODS FOR ECONOMISTS 

Time: 2.5 Hours
This paper has TWO printed pages and THREE parts
Part A: Answer any FIVE of the following:
$2 \times 5=10$

1. Given $\mathrm{A}=\left(\begin{array}{cc}7 & -1 \\ 6 & 9\end{array}\right), \mathrm{B}=\left(\begin{array}{cc}0 & 4 \\ 3 & -2\end{array}\right)$ and $\mathrm{C}=\left(\begin{array}{ll}8 & 3 \\ 6 & 1\end{array}\right)$

Find out
a. $4 \mathrm{~B}+2 \mathrm{C}$,
b. $\mathrm{C}-\mathrm{A}$
2. Test whether the following matrices are non-singular:
$A=\left(\begin{array}{ccc}4 & 0 & 1 \\ 19 & 1 & -3 \\ 7 & 1 & 0\end{array}\right), B=\left[\begin{array}{ccc}4 & 9 & 5 \\ 3 & 0 & 1 \\ 10 & 8 & 6\end{array}\right)$
What can you conclude about the rank of each matrix?
3. Given $Z=e^{2 x+2 y}$, show that $\mathrm{f}_{\mathrm{xy}}-\mathrm{f}_{\mathrm{yx}}=0$.
4. Evaluate the indefinite integral of $\int\left(x^{\wedge} 2+2 x+1\right) d x$.
5. Find the elasticity of substitution for the production function, $q=10-1 / K-1 / \mathrm{L}$.
6. Define dominant strategy.
7. Define Nash equilibrium.

Part B.Answer any THREE of the following:
$10 \times 3=30$
8. Consider the following Keynesian model with money:
$\mathrm{C}=0.8 \mathrm{Y} ; \mathrm{I}=102-0.2 \mathrm{r}, \mathrm{Md}=0.25 \mathrm{Y}-2.5 \mathrm{r}, \mathrm{Ms}=100$,
The equilibrium conditions are:
$\mathrm{Y}=\mathrm{C}+\mathrm{I}, \mathrm{Md}=\mathrm{Ms}$,
where, Y (national income), C (consumption expenditure), I (investment), Md (money demand), Ms (money supply), and r (rate of interest).
a) Write down the equations for the IS and LM curves.
b) Evaluate the equilibrium values of Y and r using Cramer's rule.
9. a) A consumer has the utility function $U=x^{\alpha} y^{\beta}$, such that $0<\alpha<1$ and $0<\beta<1$. Show that (i) there is diminishing marginal utility to increased consumption of either commodity, (ii) the indifference curves are downward sloping.
b)The demand equation of a commodity is given as $x_{1}=300-p_{1}^{2} / 2+p_{2} / 50+y / 20$, where $p_{1}$ is the price of $x, p_{2}$ is the price of a related commodity and $y$ is the income of the consumer. Find the price and income elasticity of demand for $\mathrm{x}_{1}$ when $\mathrm{p}_{1}=10, \mathrm{p}_{2}=$ 15 and $\mathrm{y}=300$.
10. Given $\mathrm{Q}=\mathrm{AK}^{\alpha} \mathrm{L}^{1-\alpha}$, verify Euler's theorem and calculate elasticity of substitution.
11. a) The demand function of a commodity is given as $p=8-x^{3}$. Find the consumer's surplus, if the commodity in question is free good.
b) Suppose, the cost function of a firm is $C=x^{2}-x y+y^{2}$. The firm is bound to produce 10 units. Find the minimum cost the firm has to incur to keep the output constant?
12. Suppose A and B both harvest fish on a big lake. They can either co-operate with each other by limiting their fishing fleet to one ship per day, or they can act non-co-operatively by sending out three ships per day. If they co-operate and send out only one ship they can each earn net profits of $\$ 300$. But if A sends out three ships while B only sends out one, A can increase his net profit to $\$ 400$ by capturing a disproportionate share of the rents. B would only earn net profit of $\$ 100$. B has the same incentive: if A co-operates and B does not, B can earn net profits of \$ 400 while A only earns $\$ 100$. If both fall for the incentive to act non-cooperatively by sending out three ships, they overfish big lake and their net profit fall to \$ 150 each. Find out the equilibrium combination.

## Part C. Answer any TWO of the following:


13. a) If the production function is of the form $\mathrm{q}=8 \mathrm{x}_{1}{ }^{1 / 2}+20 \mathrm{x}_{2}^{1 / 2}$. and if $\mathrm{r}_{1}=1, \mathrm{r}_{2}=5$, derive the equation of the expansion path.
b)Assume that the market demand is $\mathrm{P}=100-0.5 \mathrm{X}$ and the two colluding firms have costs given by $\mathrm{C}_{1}=5 \mathrm{X}_{1}$ and $\mathrm{C}_{2}=0.5 \mathrm{X}^{2}$. Find the cartel quantities and profit.
14. Let the technological coefficient matrix (A) and the final demand vector (D)are given by: The coefficient matrix, $A=\left(\begin{array}{lll}1 / 8 & 1 / 3 & 1 / 4 \\ 1 / 2 & 1 / 6 & 1 / 4 \\ 1 / 4 & 1 / 6 & 1 / 4\end{array}\right)$ and the final demand vector, $\left.D=\begin{array}{l}10 \\ 28 \\ 14\end{array}\right]$
Find the total output of the three sectors. What will be the total output if the final demand for sector 1 has increased by 1 ?
15. a) A carpenter produces two products: chairs and tables. Processing of these products is done on two machines A and B . A chair requires 2 hours on machine A and 6 hours on machine B. A table requires 5 hours on machine A and no time on machine B. There are 20 hours of time per day available on machine A and 30 hours on machine B. Profit gained by the carpenter from a chair and a table is Re. 1 and Rs. 5 respectively.Using Simplex method, find, what should be the daily production of each of the two products?
b) Solve the game whose payoff matrix is given by:

Player B
Player A $\left.\begin{array}{c} \\ A 1 \\ A 2 \\ A 3\end{array} \begin{array}{cccc}B 1 & B 2 & B 3 & B 4 \\ 3 & -1 & 4 & 2 \\ -1 & -3 & -7 & 0 \\ 4 & -6 & 2 & -9\end{array}\right)$

