Register Number:

Date:

St. Joseph's College(Autonomous), Bangalore
B.Sc Mathematics- VI Semester
Semester Examination: April,2019
MT6115: Mathematics- VII

Duration: 2.5 Hours **Max. Marks:**70

(The paper has 2 printed pages and 3 parts .)

I. Answer any Five questions. (5 x 2 = 10 marks)

1. Is $S = \{(x, y)|x + y + 1 = 0\}$ a subspace of R^2? Justify.
2. Is $A = \{(1, 0, 1), (1, 1, 1), (0, 1, 0)\}$ a basis of R^3 ? Justify.
3. Is $T : R^2 \longrightarrow R^2$ given by $T(x, y) = (-y, 2x)$ a linear transformation? Justify.
4. Find the arc element in the cylindrical polar coordinates.
5. Find the scale factors of sherical polar coordinates.
6. Form the partial differential equation by eliminating the arbitrary constants a, b from the equation $x^2 + y^2 + (z - b)^2 = a^2$.
7. Solve $q = e^{-(p/2)}$.
8. Solve $\dfrac{dx}{y - z} = \dfrac{dy}{z - x} = \dfrac{dz}{x - y}$.

II. Answer any THREE questions. (3 x 6 = 18 marks)

9. Prove that the union of two subspaces of a vector space V over a field F is a subspace of V if and only if one of the subspace is contained in the other.
10. Find the basis and dimension of the subspace spanned by the vectors $(2, 4, 2), (1, -1, 0), (1, 2, 1)$ and $(0, 3, 1)$ in R^3.
11. Given the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{pmatrix}$, find the linear transformation $T : R^2 \longrightarrow R^3$ relative to the bases $B_1 = \{(1, -1), (-1, 1)\}$ and $B_2 = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$.
12. State and Prove Rank Nullity theorem.

III. Answer any SEVEN questions. (7 x 6 = 42 marks)

13. Check for the integrability of the following Differential equation and if integrable, solve it.
$zdx + zdy + [2(x + y) + sinz]dz = 0$

14. Solve $p + q = x + y + z$

15. (a) Solve : $z = pq$
(b) Solve : $p - q = \sin x + \cos y$

16. Using Charpit's method, solve $q = (z + px)^2$

17. Solve $[D^2 - 3DD' + 2D'^2]z = x \cos y$

18. A tightly stretched string with fixed end points $x = 0$ and $x = 2$ is initially in the the position given by

$$u(x, 0) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - x, & 1 \leq x \leq 2 \end{cases}$$

If it is released from rest from this position, find the displacement $u(x, t)$.

19. Show that the spherical polar coordinates system is an orthogonal coordinate system.

20. Express the base vectors of the spherical polar coordinate system, $\hat{e_\rho}, \hat{e_\theta}$ and $\hat{e_\phi}$ in terms of the base vectors of the cylindrical polar coordinate system, $\hat{e_\rho}, \hat{e_\phi}$ and $\hat{k}$ and hence derive the expressions for $\hat{e_\rho}, \hat{e_\phi}$ and $\hat{k}$ in terms of $\hat{e_\rho}, \hat{e_\theta}$ and $\hat{e_\phi}$.

21. Express $yz\hat{i} - zx\hat{j} + xy\hat{k}$ in cylindrical polar coordinates.