Register Number:
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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - I SEMESTER

SEMESTER EXAMINATION- JANUARY 2021

PH 7120 - CLASSICAL MECHANICS

Time-2 1/2 hrs. **Maximum Marks-70**

This question paper has 2 printed pages and 2 parts

PART A

Answer any FIVE full questions. **(5x10=50)**

1. Consider a system of N particles defined in the positional coordinates system by the vectors: $\vec{r}_i$. If we analyze the system in a general coordinate system chosen in a manner so as to eliminate the j equations of constraints in the system, then the N position vectors will depend on $3N-j$ generalized coordinates $\{q_k\}$. From the definition of the Kinetic Energy of the system: $T=\sum_{i=1}^{N}\frac{1}{2}m_i\vec{r}_1\cdot\vec{r}_i$ being an invariant in the two coordinate systems, derive the expression for the Generalized Forces.
2. For a Lagrangian $L=L(q_k,\dot{q}_k,t)$ we may obtain the energy function h from the time derivative of L as $h=\sum_{k=1}^{n}\dot{q}_k\frac{\partial L}{\partial q_k}-L$. Show (in a heuristic manner) using general arguments about the Kinetic Energy $T(q_k,\dot{q}_k)$ that for most systems of interest, if L is not explicitly dependent on time and the transformations $\vec{r}_i=\vec{r}_i(\{q_k\})$ do not involve time explicitly and further if the potential $U=U(q_k)$ does not depend on the generalized velocities: $\dot{q}_k$, then h is a conserved quantity and may be identified as the total energy of the system.
3. Show that when two mass points m_1 and m_2 move about each other interacting through a potential $U=U(r,\dot{r},...)$ that depends on the magnitude of the relative positions of the particles and the magnitude of their velocities measured in the center of mass frame, the Lagrangian can be reduced to $L=\frac{1}{2}m\dot{\vec{r}}\cdot\dot{\vec{r}}-U(r,\dot{r},...)$.

4. Write down the Lagrangian (for a particle of reduced mass μ) moving in a potential corresponding to an inverse square law force and from this Lagrangian, obtain the Hamiltonian.
5. Using variations of parametrized paths in the configuration space, and taking the Hamiltonian to be a constant in time, show that the action integral:
$$\int_{t_1}^{t_2} L\,dt=\int_{t_1}^{t_2}\left(\sum_{k=1}^{n} p_k \dot{q}_k\right)dt-\int_{t_1}^{t_2} H\,dt$$
leads to the Least Action Principle.
6. The waves on a string are represented by the wave equation: $\frac{\partial^2 y}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$. By introducing the variables $u=x-ct$ and $v=x+ct$ obtain the d'Alembert solution to the wave equation.
7. With a neat diagram displaying Rotational Transformation of coordinates, show that the Generalized Force is in the direction of the net torque.

PART B

Answer any <u>FOUR</u> full questions. (4x5=20)

8. A bead of mass m is sliding down a wire kept at an angle α with respect to the horizontal. Show that the virtual work done on the bead by the constraint forces vanish.
9. A bead of mass m is constrained to move on a vertical circle. Obtain the equation of motion of the bead using the Lagrangian method.
10. Using the principle of the calculus of variations, find the extremum of the integral:
$$I=\int_a^b \left[9x+y+\sqrt{\frac{\partial y}{\partial x}}\right]dx\ .$$
11. We know that the magnitude of the apsidal distance of a particle in an elliptical orbit is given as: $r=\frac{a(1-e^2)}{1+e\cos\phi}$. The Halley comet has an eccentricity of $e=0.967$ at its closest approach (perigee), the comet is at a distance of $0.59\ \mathrm{AU}$ from the Sun (AU stands for the measure of the average distance between the Earth and Sun in a year and is equal to $1.5\times10^8\ \mathrm{km}$). What is the farthest it goes (i.e. what is its apogee) from the Sun assuming a perfectly elliptical orbit?
12. A bead of mass m is constrained to move on a horizontal circle. Obtain the equation of motion of the bead using the Hamiltonian method (the Hamiltonian has to be clearly obtained and expressed so, if your answer needs to be adjudged).
13. Using the d'Alembert solution, solve the wave equation with $c=4$ for a finite string of length $L=\pi$ with the periodic boundary conditions that $y(0,t)=y(\pi,t)=0$ and initial conditions that $y(x,0)=\sin x$ and $\frac{\partial y}{\partial t}(x,0)=\sin x$.