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Register Number:

DATE:

**ST. JOSEPH’S COLLEGE (AUTONOMOUS), BANGALORE-27**

**B.Sc. MATHEMATICS – VI SEMESTER**

**SUPPLEMENTARY EXAMINATION: APRIL 2019**

MT 6112: Mathematics Paper 7

Time-3 hrs Max Marks-100

**This paper contains THREE printed pages and FOUR parts.**

1. **ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS: (8 X 2 =16)**
2. Evaluate  along the curve $y=x^{2}+1$ from $\left(0, 1\right)$ to $\left(3, 10\right)$.
3. Show that  where c is the curve 
4. Evaluate 
5. Sketch the region of integration  for  and write an equivalent integral with the order of integration reversed.
6. Find volume of unit cube using triple integration.
7. Evaluate using Green’s theorem or otherwise where ** is the unit circle.
8. Define a Topological space.
9. If. Is 2, an interior point a of?
10. Is the additive group of integers a vector space over the field of real numbers? Justify.
11. Is  is a subspace of the vector space ? Justify.
12. Define Basis and Dimension of a vector space.
13. Define Linear transformation.
14. **ANSWER ANY SEVEN OF THE FOLLOWING QUESTIONS: (7 X 6 = 42)**
15. Show that.
16. Evaluate 
17. Evaluate  over the annular region between the circles x2 + y2 = 4 and x2 + y2 = 1.
18. Evaluate where is the triangle formed by vertices.
19. Find the area bounded by the curves  and.
20. Find by double integration the area which lies inside the Cardioid  and outside the circle r = a.
21. Find the volume of tetrahedron bounded by the coordinate planes and.
22. State and prove Green’s theorem in the plane.
23. Using Gauss’ divergence theorem evaluate  where S is
 the surface of the cube bounded by the planes. 
24. State and Prove Stokes’ theorem.
25. **ANSWER ANY TWO OF THE FOLLOWING QUESTIONS: (2 X 6 =12)**
26. If is a topological space then prove that
27. Any intersection of closed sets is closed.
28. Union of any two closed sets is closed.
29. A set is open if and only if it is a neighbourhood of each of its points.
30. Define Basis and subbasis of a topological space X.
If X = { 1, 2, 3, 4 } and A = {{1, 3}, {2, 4}, {4}}. Find the corresponding basis and the Topology generated by A.
31. **ANSWER ANY FIVE OF THE FOLLOWING QUESTIONS : (5 X 6=30)**
32. (i) Define Subspace.

(ii) Prove that a non-empty subset W of a vector space  is a subspace if and

 only if and .

1. An ordered set not all zero, of a vector space  with , is

 linearly dependent if and only if one of the vectors say is a linear

 combination of its preceding vectors.

1. Let be the field of integers modulo 3. Find a basis and the dimension of the subspace of  spanned by.
2. Verify whether the following set of vectors { ( 1,6,4 ) ( 2,4,-1 ) ( -1,2,5 )} form a basis of
V 3 ( R ). If not find a basis and dimension of the subspace spanned by these vectors .
3. Find the linear transformation  such that  is the matrix of  relative to the base.
4. (i) Define Range space and Null space of a linear transformation.

(ii) If is a linear transformation, then prove that the range spaceis a subspace of and the null space is a subspace of .

1. State and Prove rank – nullity theorem for vector spaces.