Register Number:

DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

B.Sc. MATHEMATICS – VI SEMESTER

SEMESTER EXAMINATION: APRIL 2018

MT 6115 : MATHEMATICS - VII

Time- 2 ½ hrs **Max Marks-70**

This question paper has three parts two printed pages.

I. Answer any five from the following **5 X 2 =10**

1. Verify the condition for integrability for $(y^2+z^2-x^2)dx-2xydy-2xzdz=0$
2. Form the partial differential equation by eliminating arbitrary constants a and b from $z=(x^2+a)\,(y^2+b)$.
3. Solve $p+q=sinx+siny$
4. Find the general solution of $(D-D')^2\,z=e^{x+y}$.
5. Is the subset $W=\{(x_1,x_2,x_3)/\,{x_1}^2+{x_2}^2+{x_3}^2\leq 0\}$ of the vector space $V_3(R)$is a subspace of $V_3(R)$? Justify.
6. In a n dimensional vector space $V(F)$, prove that any $n+1$ elements of $V$are linearly dependent.
7. Find the matrix of the linear transformation $T\colon V_3(R)\to V_2(R)$ defined by $T(x,y,z)=(x+y,y+z)$ relative to the standard bases of $V_3(R)\ and\ V_2(R)$.
8. Find the scalar factors $h_1,h_2\,,h_3$ for cylindrical polar coordinates.

II. Answer any seven from the following **7 X 6 =42**

9. Solve $z^2dx+(z^2-2yz)dy+(2y^2-yz-zx)dz=0$.
10. Solve $z(x+y)p+z(x-y)q=x^2+y^2$.
11. Solve $x^2p^2+y^2q^2=z^2$ using the transformation $u=logx\,,v=logy\,,w=logz$
12. Solve $p(1+q)=zq$.
13. Find a complete integral of $2xz-px^2-2qxy+pq=0$ using Charpit's method.
14. Find the general solution of $\left(D^2-2DD'+D'^2\right)z=12xy$.

15. Derive the Fourier series solution of one dimensional heat equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

16. Derive the arc element and volume element for spherical coordinates.

17. Derive e_ρ , e_φ , e_z in terms of i , j , k and then find i , j , k in terms of e_ρ , e_φ , e_z .

III. Answer any three from the following **3 X 6=18**

18 Prove that the union of any two subspaces of a vector space V over a field F is a subspace if and only if one is contained in the other.

19 Prove that if S is a nonempty subset of a vector space $V(F)$ then

(i) $L(S)$ is a subspace of V.

(ii) $L(S)$ is the smallest subspace of Vcontaining S .

20 Given the matrix $A=\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$ find the linear transformation $T : V_2(R) \to V_3(R)$ relative to the bases $B_1 = \{(1,1), (-1,1)\}$ and $B_2 = \{(1,1,1), (1,-1,1), (0,0,1)\}$

21 If T is a linear transformation from $V_3(R)$ $into$ $V_4(R)$ defined by $T(1,0,0) = (0,1,0,2)$, $T(0,1,0,) = (0,1,1,0)$, $T(0,0,1) = (0,1,-1,4)$. Find the range space, null space, rank and nullity of T .