Register Number: Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

B.Sc. MATHEMATICS – VI SEMESTER

SEMESTER EXAMINATION: APRIL 2018

MT 6215 MATHEMATICS - VIII

Time: 2 ½ hrs **Max Marks: 70**

This paper contains TWO printed pages and THREE parts.

I Answer any FIVE of the following. **5 X 2 = 10**

1. Evaluate: $\lim_{z\to e^{\frac{i\pi}{2}}} \frac{z^3+z^2+1+i}{z^6+1}$.

2. Define (i) Analytic function (ii) Harmonic function.

3. If $f(z)=z^2$, where $z=x+iy$, show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=16|z|^2$.

4. Evaluate: $\int_C \frac{z^2}{z+1-i}dz$, where C is the circle $|z|=\frac{3}{2}$.

5. Find the fixed points of the bilinear transformation $w=\frac{z-9}{z+1}$.

6. Using Newton-Raphson method find x_1, given that $x\log_{10}x=12.34$ and $x_0=10$.

7. Evaluate: $L\,[10^t(1-2\sin^2 5t)]$.

8. Find the inverse Laplace transform of $\frac{s+1}{s(s+2)-1}$.

II Answer any SEVEN of the following. **7 x 6 = 42**

9. If z is a complex variable, show that the locus of the point z satisfying the relation $|z+1|+|z-1|=4$ is an ellipse and find its eccentricity.

10. State and prove the sufficient conditions for a function $f(z) = u + iv$ to be analytic in a domain D.

11. Show that $u = \log\sqrt{x^2 + y^2}$ is harmonic and find its harmonic conjugate.

12. Find the real part of the analytic function whose imaginary part is $\left(\mathrm{r} - \frac{1}{\mathrm{r}}\right)\sin\theta$.

13. Evaluate: $\int_C \frac{\cos \pi z}{e^{2z}(z+1)^2(z-2)} dz$, where C is the ellipse $9x^2 + 4y^2 = 36$.

14. (i) If a complex function $f(z)$ is analytic within and on the circle $C: |z-a| = r$, then prove that

$\left|f^{(n)}(a)\right| \le \frac{Mn!}{r^n}$, where M is the maximum value of $|f(z)|$ on C *and* $n = 0,1,2,...$

(ii) If $f(z)$ is analytic and bounded for all z in the entire complex plane, prove that $f(z)$ is constant.

15. Discuss the transformation $w = z^2$.

16. Define: (i) Bilinear transformation (ii) The cross ratio of four points.

Prove that a bilinear transformation preserves the cross ratio of four points.

17. Find the bilinear transformation which maps $z = -1, 1, \infty$ onto $w = -i, -1, i$.

III Answer any THREE of the following. 3 x 6 = 18

18. Solve the initial value problem $y' = y + x,\ y(0) = 1$, using the modified Euler's method with

$h = 0.05$ for $x \in [0, 0.1]$.

19. Evaluate: (i) $\int_0^\infty \frac{e^{-t}\sin t}{t} dt$ (ii) $L^{-1}[\log(s+1)]$.

20. Define Heaviside function $H(t-a)$. Express $f(t) = \begin{cases} t^2, & 0 < t < \pi \\ \pi t, & t > \pi \end{cases}$ in terms of Heaviside

function and then find $L[f(t)]$.

21. Find $f(t)$ from the equation $f'(t) = t + \int_0^t f(t-u)\cos u\, du$, given $f(0) = 4$.