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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - III SEMESTER

SEMESTER EXAMINATION- NOVEMBER 2020

PH 9118 - QUANTUM MECHANICS II

Time-2 1/2 hrs. **Maximum Marks-70**

This question paper has 4 printed pages and 2 parts and includes a table of constants and integrals

PART A

Answer any FIVE full questions. **(5x10=50)**

1. Spin 1/2 particles can be represented by two states: the up state $|1/2,+1/2\rangle=\begin{pmatrix}1\\0\end{pmatrix}$, and the down state $|1/2,-1/2\rangle=\begin{pmatrix}0\\1\end{pmatrix}$ where the notation on the left hand side is the ket that is labelled in the form of $|s,m\rangle$; s is the spin quantum number and m the magnetic quantum number (since the operators $\hat{S}^2$ and $\hat{S}_z$ form a complete set of commuting observables). We also use the vector notation of $\chi_+=\begin{pmatrix}1\\0\end{pmatrix}$ and $\chi_-=\begin{pmatrix}0\\1\end{pmatrix}$ which form an orthonormal basis in the spin-1/2 space. Using the eigenvalue equations for the operators $\hat{S}^2$ and $\hat{S}_z$: and obtain the matrix form for the operators $\hat{S}^2$ and $\hat{S}_z$.
2. The ground state wave function for the hydrogen atom is given as $\psi_{100}(r,\theta,\phi)=R_{10}(r)Y_0^0(\theta,\phi)$. The radial solution is: $R_{10}(r)=\frac{a_0}{a}e^{-r/a}$ and the angular component is: $Y_0^0(\theta,\phi)=\frac{1}{\sqrt{4\pi}}$. Normalize ψ_{100} (remember: it's spherical polar coordinates).
3.
   (a) Starting with the time dependent Schrodinger equation , express the evolution equation of the components of the basis vectors (i.e. derive the equation:

$$i\hbar\frac{d c_k}{dt}=E_k c_k(t)+\sum_n \lambda c_n(t)\langle \phi_k|\hat{W}|\phi_n\rangle \quad .$$

From this equation show that $c_k(t)=b_k e^{-iE_k t/\hbar}$. Express the time dependent perturbation equation in terms of b_n .

(b) For a system subject to a time dependent perturbation, write down an expression for transition probability from an initial state $|\phi_i\rangle$, to a final state $|\phi_f\rangle$. (5+5)

4. A particle in an infinite potential, containing three levels with $n=1$, $n=2$ and $n=3$ is subjected to a time independent perturbation due to a potential: $W=10^{-3}V_0\frac{x^2}{L^2}$ where x is the position of the particle and L is the length of the box and V_0 is a constant with dimensions of energy. Estimate the first order change to all the 3 energy levels.

5.
(a) In the region where the WKB approximation fails, express the potential as a linear function of the form $V(x)\approx E+V'(0)x$. Substitute in Schrodinger equation to obtain the Airy Equation.
(b) Write down the general form for the wave function in the overlap region of the classical and non-classical region. (5+5)

6.
(a) What are the possible states for a system composed of two spin half particles? Construct permutation operators for these.
(b) Based on the states derived above, construct completely symmetric and completely antisymmetric states. Use the permutation operators constructed in section (a) to show the symmetry operations. (5+5)

7. For a particle in a box potential defined between $0\leq x\leq L$, use the Variational Method to
(a) obtain the ground state energy
(b) compute the percentage difference in the ground state energy

You may use the trial wavefuction: $\psi_{\text{trial}}(x)=A x^2(x-L)^2$ (8+2)

PART B

Answer any <u>FOUR</u> full questions. <u>(4x5=20)</u>

8. For a system consisting of a particle with $j_1=1$ and another with $j_2=2$, what are **all** the possible states of the composite system and how will they be related to the individual states of each of the particles? You **do not** have to calculate the Clebsch-Gordan coefficients and also, you do not need to ortho-normalize the states.

9. Consider a quantum system with just three linearly independent states. The Hamiltonian, in matrix form, is: $H=V_0\begin{pmatrix}(1-\epsilon) & 0 & 0\\ 0 & 1 & \epsilon\\ 0 & \epsilon & 2\end{pmatrix}$ where V_0 is a constant and ϵ is some small number ($\epsilon\ll 1$)
(a) Write down the eigenvectors and eigenvalues of the unperturbed Hamiltonian ($\epsilon=0$). Identify the degenerate and the nondegenerate eigenstates
(b) Solve for the exact eigenvalues of H .
(c) Use first order **non-degenerate perturbation theory** to find the approximate eigenvalue

for the state that evolves out of the nondegenerate eigenvector of H_0 (1+2+2)

10. Permutation of three particles $(a,b,c)\rightarrow(b,c,a)$ maybe represented by the matrix operation

$$\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}b\\c\\a\end{pmatrix} .$$

(a) In a similar manner write down a 4x4 matrix representation for a permutation of four objects $(a,b,c,d)\rightarrow(d,a,c,b)$.

(b) Write down the matrices A and B for two consecutive permutations $(a,b,c,d)\rightarrow(a,b,d,c)\rightarrow(d,a,c,b)$.

(c) Do the two matrices in (b) above commute (i.e. is $AB=BA$)? (1+1+3)

11. Obtain the first order change to the ground state level eigenfunction for Question 4 in Part A.

12. When a system in eigenstates $|\phi_i\rangle$ are subjected to a time-dependent perturbation, the components of eigenstates evolve with time through the integral (where the system is assumed to attain an eigen state $|\phi_k\rangle$ in time t):

$b_k^{(1)}(t)=\frac{\lambda}{i\hbar}\int_0^t e^{i\omega_{ki}t'}\langle\phi_k|\hat{W}|\phi_i\rangle dt'$. If the final states are continuum states with density $\rho(\beta,E)$, obtain the expression for transition probability (Fermi Golden Rule).

13. For a Hydrogen atom placed in a constant electric field ε , obtain an expression for the perturbed Hamiltonian. Using this, derive the first order perturbation to the ground state Energy. The wavefunction for the ground state of Hydrogen atom is:

$$\psi_{100}(r,\theta,\phi)=\frac{1}{\sqrt{\pi a^3}}e^{-r/a}$$

(Some) Physical Constants

[Constants: **h**=6.626070x10^{-34} J s (**Planck's constant**), **1eV** = 1.6x10^{-19} J (**electron volt to Joules**), c=2.99792458x10^{8} m/s (**speed of light**), **1Å** = 1x10^{-10}m (**Angstrom to meters**), **e** = 1.602176x10^{-19} C (**electronic charge**), ε_0= 8.85418782x10^{-12}m^{-3}kg^{-1}s^{4}A^{2} (**permittivity of free space**), m_{proton}=1.672621898x10^{-27}kg (**mass of proton**), $m_{electron}$=9.10938356x10^{-31}kg (**mass of electron**), $m_{neutron}$=1.674927471x10^{-27}kg (**mass of neutron**), **a** = 5.029x10^{-10}m (**Bohr radius**), **α** = 1/137 (**Fine Structure constant**), G=6.674x10^{-11}m^{3}kg^{-1}s^{-2} (**Gravitational constant**), $M_\odot$=1.9891x10^{30} kg (**Solar mass**), $R_\odot$=6.9x10^{8} m (**Sun's Radius**), **σ** = 5.67x10^{-8} W m^{-2} K^{-4} (**Stefan-Boltzmann constant**), M_{Earth}=5.97x10^{27}kg (**Mass of Earth**), $D_{earth\text{-}sun}$=1.49x10^{11}m (**Earth-Sun distance**), 1 inch = 2.54 cm, 1foot=12 inches]

(a) Table of (some) Integrals

Gamma Function:
$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$
$\Gamma(n)=(n-1)!$
$\Gamma(\frac{1}{2}+n)=\frac{(2n)!}{4^n n!}\sqrt{\pi}$

(a) $\int_0^\infty e^{-2bt}dt=\frac{1}{2b}$

(b) $\int_0^\infty t\,e^{-2bt}dt=\frac{1}{4b^2}$

(c) $\int_0^\infty t^2 e^{-2bt}dt=\frac{1}{4b^3}$

(d) $\int_0^\infty t^3 e^{-2bt}dt=\frac{3}{8b^4}$

(e) $\int_0^\infty t^4 e^{-2bt}dt=\frac{3}{4b^5}$

(f) $\int_0^\infty t^5 e^{-2bt}dt=\frac{15}{8b^6}$

(g) $\int_0^\infty t^6 e^{-2bt}dt=\frac{45}{8b^7}$

(h) $\int \frac{1}{t^2+b^2}dt=\frac{1}{b}\tan^{-1}\left(\frac{t}{b}\right)$

(i) $\int \frac{1}{(t^2+b^2)^2}dt=\frac{1}{2b^3}\left(\frac{bt}{(b^2+t^2)}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(j) $\int \frac{1}{(t^2+b^2)^4}dt=\frac{1}{16b^7}\left(\frac{15t^5b+40b^3 5^3+33b^5t}{(3t^6+9bt^4+9b^3t^2+b^5)}+5\tan^{-1}\left(\frac{t}{b}\right)\right)$

(k) $\int \frac{t^2}{(t^2+b^2)^2}dt=\left(-\frac{t}{(2b^2+2t^2)}+\frac{1}{2b}\tan^{-1}\left(\frac{t}{b}\right)\right)$

(l) $\int \frac{1}{(t^2+b^2)^3}dt=\frac{3}{8b^5}\left(\frac{5/3b^3t+bt^3}{(b^2+t^2)^2}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(m) $\int \frac{t^2}{(t^2+b^2)^4}dt=\frac{1}{16b^5}\left(\frac{bt^5+8/3b^3t^3-3b^5t}{(b^2+t^2)^3}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(n) $\int \sqrt{a/x-1}\,dx=x\sqrt{a/x-1}+a\ \tan^{-1}\left(\sqrt{a/x-1}\right)$

(o) $\int \sqrt{1-ax}\,dx=-\frac{2(1-ax)^{3/2}}{3a}$

(p) $\int \sqrt{1-ax^2}\,dx=\frac{1}{2}x\sqrt{1-ax^2}+\frac{\sin^{-1}\sqrt{a}x}{2\sqrt{a}}$

(q) $\int_{-\infty}^{\infty} e^{-\alpha t^2+i\omega t}dt=\sqrt{\frac{\pi}{\alpha}}e^{-\frac{\omega^2}{4\alpha}}$

(r) $\int_0^\infty t^4 e^{-\alpha^2 t^2}dt=\frac{3\sqrt{\pi}}{8\alpha^5}$

(s) $\int_0^\infty t^n e^{-st}dt=\frac{n!}{s^{n+1}}$ (Laplace Transform)

PH9118_A_20