**ST. JOSEPH’S COLLEGE (AUTONOMOUS), BANGALORE-27**

DATE:**6-04-2016**

B.Sc Mathematics: VI SEMESTER

SEMESTER EXAMINATION- APRIL 2018

**­­­­MT 6112: Mathematics Paper - VII**

**Time: 3hrs Maximum marks: 100**

(For supplementary candidates)

Do not write the register number on the question paper

Please attach the question paper along with the answer script.

 This question paper has three printed pages and four parts.

1. **ANSWER ANY EIGHT OF THE FOLLOWING **
2. Show that  is independent of the path joining andand hence evaluate.
3. Find the area bounded by one loop of the lemniscate
4. Find the area of the surface 
5. If C is a simple closed curve in the xy- plane, prove by Green’s theorem that the integral  represents the area A enclosed by C.
6. If S is the surface of the sphere  enclosing a volume V and

where a, b, c are constants. Prove that 

1. Evaluate by Stokes theorem: where C is the curve
2. Write all possible topologies for $X=\{2,3\}$.
3. Show that every convergent sequence is a Cauchy sequence.
4. Write the vector $∝=\left( 1, 7, -4 \right)$ as linear combination of vectors  in vector space 
5. Show that the vectors $\left\{1,2,1\right\},\left(2,1,0\right),\left(1,-1,2\right)\}$ form a basis of .

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1. If  defined by . Examine whether T is a linear transformation.
2. Find the matrix of the linear transformation  defined by  with respect to a standard basis.
3. **ANSWER ANY SEVEN OF THE FOLLOWING **
4. Evaluate  around the boundary of the region defined by

.

1. Evaluate  by changing the order of integration.
2. By changing the variables appropriately evaluate  over the positive quadrant bounded by the circle .
3. Find by double integration the area which lies inside the Cardioid  and outside the circle r = a.
4. Find the volume bounded by the cylinder  and the planes $y+z=3 , z=0.$
5. State and prove Greens theorem in the plane.
6. Verify Greens theorem for  where C is the boundary of the region enclosed by the lines $x=0,y=0,x+y=1$
7. Using Divergence theorem, evaluate  where S is the closed surface bounded by the cone  and the plane $z=1.$
8. Verify Stokes theorem for  where S is the upper half surface of the sphere  and C is its boundary.
9. Find the surface area of the sphere .
10. **ANSWER ANY TWO OF THE FOLLOWING **
11. Prove that the intersection of any two open discs is also an open set.
12. i) Define Topological space. ii) If $X=\left\{a,b,c\right\}$and $τ=\{X,∅,\left\{a\right\},\left\{b\right\},\left\{a,b\right\}\}$then prove that $τ$is a topological space.
13. i) Define interior points. ii) If $X=\{a,b,c\}$and $τ=\{X,∅,\left\{b\right\},\left\{b,c\right\},\left\{a,b\right\}\}$is a topology for X, then determine the interior of $A=\{a,c\}$and interior of B$=\{b,c\}$.
14. **ANSWER ANY FIVE OF THE FOLLOWING **
15. Define a subspace and hence prove that the intersection of any two subspaces of a vector space V over a field F is also a subspace of V. Does this property hold for union of two subspaces. Justify your answer.
16. Find the basis and dimension of the subspace spanned by the vectors

 of 

1. Find the linear transformation such thatand 
2. Prove that every linearly independent subset of a finitely generated vector space V(F) forms a part of a basis of V.
3. Prove that if a non empty subset W is a subspace of a vector space V over F if and only if 
4. Find the matrix of the linear transformation  defined by

 relative to bases 



1.  is defined by . Find the range space, null space, rank, nullity and hence verify the rank nullity theorem.

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