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Register Number:

DATE: 11-04-2017

**ST. JOSEPH’S COLLEGE (AUTONOMOUS), BENGALURU-27**

**M.Sc. MATHEMATICS- IV SEMESTER**

**SEMESTER EXAMINATION- APRIL 2017**

**MT 0114: Measure and Integration**

**Time:** 2 ½ **Hours Max. Marks: 70**

**This question paper has two printed pages.**

**Answer Any SEVEN Questions**1**.** i) Define an outer measure. (1)  
 ii) Prove that outer measure is monotone (2)  
 iii) Prove that a countable set has outer measure zero. (3)  
 iv) Prove that the outer measure is countably sub additive (4)

2. i) Prove that a countable set has outer measure zero. (3)  
 ii) If is a descending collection of measurable sets and m(B1)   
 then prove that . (7)

3. i) Prove that the interval (a , is measurable for every a . (7)  
 ii) State Littlewood's three principles (3)

4. i) Define measurable function (2)  
 ii) If f and g are two measurable functions defined on the same domain, prove that   
 f + g , f + c , and fg are measurable where c is a constant (8)

5. i) State and prove Fatou's lemma (7)  
 ii) Prove monotone convergence theorem (3)

6. i) Let be a finite disjoint collection of measurable subsets of the set of finite measure.  
 For , let ak  be a real number. If  on E, then prove that   
 . (4)

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ii) Let  and be simple functions defined on a set of finite measure E. Then for any α and β,  
 prove that . (4)  
 iii) If    on E then prove that  (2)

7. State and prove Vitali's Covering lemma (10)

8. State and prove Lebesgue Dominated Convergence Theorem (10)

9. Prove that a function f is an indefinite integral if and only if it is absolutely continuous (10)

10. Prove that LP spaces (1  p <) are complete (10)