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Register Number:

DATE:

**ST. JOSEPH’S COLLEGE (AUTONOMOUS), BANGALORE-27**

**M.Sc. MATHEMATICS – II SEMESTER**

**SEMESTER EXAMINATION: APRIL 2017**

**MT 8114 Algebra - II**

Time: 2 ½ hrs Max Marks: 70

**This paper contains TWO printed pages. All questions carry equal marks.**

**Answer any SEVEN of the following. 7 X 10 = 70**

***Note: In questions 1 to 5, A and B are commutative rings with unity.***

1. (a) Define “Jacobson radical”. Prove that x is an element of a Jacobson radical of A if

 and only if is a unit in A, .

 (b) If are prime ideals of A and is an ideal contained in , then prove

 that .

 2. (a) If are the ideals of A with , then prove that .

 (b) Define “Module”. If are A-modules, prove that .

3. (a) Prove that M is a finitely generated A-module if and only if M is isomorphic to a

 quotient of , for some integer .

 (b) Prove that a sequence of A-modules and A-homomorphisms 

 is exact if and only if for all A-modules M, the sequence

  is exact.

4. (a) Define “Noetherian and Artinian modules”. Prove that an A-module M is Artinian if and

 only if every nonempty collection of submodules of M has a minimal element.

 (b) Prove that an A-module M has a composition series if and only if M is both Noetherian

 and Artinian module.

5. State and prove Hilbert Basis Theorem.

6. Define “Algebraic element”. Suppose K is an extension of a field F, prove that an element

 is algebraic over F if and only if is a finite extension of F, and is equal to

 the degree of minimal polynomial of over F.

7. (a) Find the minimal polynomial of over .

 (b) If is an irreducible polynomial of degree over a field F, then prove that there

 exists an extension E of F, such that , in which has a root.

8. (a) If a polynomial  has two splitting fields over a field F, then prove that

 there exists an isomorphism of which leaves every element of F fixed.

 (b) Show that it is impossible to trisect with straight edge and compass alone.

9. (a) If are algebraic elements over a field F of characteristic 0, then prove that there

 exists an element such that .

 (b) Suppose K is a normal extension of a field F and H is a subgroup of , then

 prove that  and , where is the fixed

 field of H.

10. Prove that K is a normal extension of a field F if and only if K is the splitting field of some

 polynomial over F.