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Register Number:

DATE:

**ST. JOSEPH’S COLLEGE (AUTONOMOUS), BANGALORE-27**

**M.Sc. MATHEMATICS – II SEMESTER**

**SEMESTER EXAMINATION: APRIL 2017**

MT 8314: Functional Analysis

Time- 2 ½ hrs Max Marks-70

This question paper contains ONE printed page

**Answer any SEVEN questions**

1. i. The Supremum norm on C[0,1] is a norm.
2. Prove that C[0, 1] in a Banach space. [5 + 5]
3. State and prove Minkowoski’ inequality. [10]
4. If N is Banach space, then prove that B(N) is also a Banach space. [10]
5. Let M be a linear subspace of a normed linear space N. Let f be a functional on M. If x0 ∉ M, x0 ∈ N and if M0 = M + [x0] is the linear subspace spanned by M and x0. Then prove that for N, real normed linear space, f can be extended to a functional on M0 such that  .

[10]

1. State and prove the uniform boundedness theorem. [10]
2. i. Define an inner product space and Hilbert space.

ii. Show that  is a Hilbert space with the inner product defined as

. Where . Where norm of an element is defined by square root of inner product with itself. [10]

1. State and prove Schwarz’ inequality and hence derive the Bessel’s inequality. [10]
2. Prove that: A norm on a linear space X is induced by an inner product on it if and only if the norm satisfies the parallelogram law. If it is so, then the inner product is given by the polarization identity. [10]
3. Prove that if M is a proper subspace of H, then there exists a nonzero vector z0 in H such that z0 ⊥ M. Hence deduce that, If M is closed linear subspace of a Hilbert space H.

Then H = M + M⊥. [10]

1. i. Define a conjugate space H\* for a Hilbert space H.

ii. State and Prove Riesz representation theorem.

iii. Prove that H\* is also a Hilbert space. [2 + 4 + 4]