



Register Number:

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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
B.Sc. MATHEMATICS - I SEMESTER
SEMESTER EXAMINATION: JANUARY 2021
MT 118 – MATHEMATICS-I

Time- 2 1/2 hrs

Max Marks-70

This paper contains two printed pages and four parts

I. Answer any five of the following: (5X2=10)

1. Define Rank of the matrix.
2. Find the eigen values of $A = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$
3. Evaluate $D^n(e^x \sin x \cos x)$
4. Find the total differential of $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$
5. Evaluate $\int_0^1 x^{\frac{3}{2}}(1-x)^{\frac{3}{2}} dx$
6. Find the angle between two planes whose equations are given by $2x - y + z = 6$ and $x + y + 2z = 3$.
7. Find the radius and center of the Sphere $16x^2 + 16y^2 + 16z^2 - 16x - 8y - 16z - 35 = 0$
8. Check whether the spheres $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ cut orthogonally.

II. Answer any three of the following: (3X6=18)

9. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ by reducing to normal form.

10. Solve the following system of equations.

$$\begin{aligned}x + 2y - 5z &= -13 \\3x - y + 2z &= 1 \\2x - 2y + 3z &= 2 \\x - y + z &= -1\end{aligned}$$

11. Check whether the given matrix $A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is diagonalizable, if diagonalizable find the matrix P such that $P^{-1}AP = D$.

12. Balance the chemical equation, using linear systems.
 $HCl + Na_3PO_4 \rightarrow H_3PO_4 + NaCl$

III. Answer any five of the following:

(5X6=30)

13. Find the nth derivative of (a) $\frac{x^2}{(x+1)(x+2)}$ (b) $\cosh 3x \cdot \sin 4x$ (4+2)

14. If $y = (x + \sqrt{1+x^2})^m$ then show that
 $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

15. (a) If $u = f(y-z, z-x, x-y)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

(b) If $u = (\theta + \varphi), v = \varphi(1 + \theta)$ show that $\frac{\partial(u,v)}{\partial(\theta,\varphi)} = 1 + \theta + \varphi$ (4+2)

16. State and prove Euler's theorem and its extension for homogeneous functions.

17. If $u = \tan^{-1} \left(\frac{x^3+y^3}{x-y} \right)$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

18. Derive the reduction formula for $\int \sin^m x \cos^n x dx$

19. Evaluate $\int_0^1 \frac{x^{a-1}}{\log x} dx$, where a is a parameter, by applying differentiation under integral sign.

IV. Answer any two of the following:

(2X6=12)

20. (a) Find the angle between the line $\vec{r} = (2\hat{i} + 4\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$ and the plane $x + 2y - z = 2$.

(b) Find the equation of the planes bisecting the angle between the planes $x + 2y - 2z - 9 = 0, 3x - 4y + 12z - 26 = 0$ (2+4)

21. Find the equation of the sphere passing through $(1,0,0), (0,1,0)$ and $(0,0,1)$ and has its center on the plane $x + y + z = 6$.

22. Find the equation of the right circular cone whose vertex is at the origin, axis is $x = y = z$ and semi vertical angle is 45° .