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## ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27 <br> M.Sc. PHYSICS - III SEMESTER <br> SEMESTER EXAMINATION : OCTOBER 2021 <br> (Examination conducted in January-March 2022) <br> PH9320/PH 9318: MODERN OPTICS

Time: $\mathbf{2}$ hours 30 min .
Max Marks:70
This paper contains 2 parts and 3 printed pages.

## Part-A

Answer any 5 questions. Each carries 10 marks.
( $5 \times 10=50$ )

1. A stable laser cavity is made using two curved mirrors as shown in figure. The first mirror has a radius of curvature $\mathrm{R}_{1}$ and the second one a radius of curvature $\mathrm{R}_{2}$ with d as the separation distance between them as shown in figure. Find the ABCD matrix for the round trip within this cavity beginning by translation from the second mirror to the first mirror.
Using the condition for stable cavity as obtained from Sylvester's theorem as
$-1<(A+D)<1$, show that the condition for the stable laser cavity in terms of $d, R_{1}, R_{2}$ is given as $0 \leq\left(1-d / R_{1}\right)\left(1-d / R_{2}\right) \leq 1$.

2. A monochromatic plane wave propagating in x -direction is incident on an opaque diffracting aperture. The optical disturbance at P from an arbitrary aperture under far-field approximation is given as $\widetilde{E}_{p}=\frac{\epsilon_{A} e^{i(\omega t-k R)}}{R} \iint_{A p} e^{i k(Y y+Z z) / R} d S$. Considering the specific rectangular configuration of the aperture as shown in the figure, find the resultant intensity at P. Comment on what kind of diffraction pattern would one obtain in this case?

3. Find the Fourier transform of the function $f(x)=\delta[x-(+d / 2)]-\delta[x-(-d / 2)]$ using the given sifting property of delta function $\int_{-\infty}^{+\infty} \delta\left(x-x_{o}\right) f(x) d x=f\left(x_{o}\right)$. Compare the result
with the calculated Fourier transform of $E(x)=\operatorname{Cos} k_{p} x \quad|x|<L, 0 \quad|x|>0$. Draw graphs for both the functions and their transforms and interpret your results.
4. What is spatial coherence? Using Young's double-hole experiment, prove that the spatial coherence is lost if an extended source has linear width $l=\frac{\lambda a}{d}$ where $\lambda$ is the wavelength of the source, $d$ is the separation distance between the two holes and 'a' is the perpendicular distance between the extended source and the aperture containing the two holes. (Hint: Start with another source $S$ ' at a distance $l$ from the $S$ and then replace it with an extended source.)
5. Consider two orthogonal optical disturbances represented as $\vec{E}_{x}=E_{o x} \cos (k z-\omega t) \hat{i}$
$\vec{E}_{y}=E_{\text {oy }} \cos (k z-\omega t+\epsilon) \hat{j}$ where $\epsilon$ is the relative phase difference between the light waves which are travelling in $z$ - direction. Eliminating the $(k z-\omega t)$ dependence from the scalar form of these waves, obtain a general equation for ellipse to show that superposition of such waves produces elliptically polarized light. Hence, show that circularly polarized light is a special case of elliptically polarized light.
6. a) Assuming that the polarization within a non-linear medium consists of both linear and non-linear terms, use Maxwell's equations in a dielectric medium and arrive at the inhomogeneous wave equation (in terms of the field $\vec{E}$ and non-linear polarization term $\vec{P}^{N L}$ ) neglecting the anisotropy of the medium.
b) Assume that an optical wave of frequency $\omega$ travelling in z-direction incident on a nonlinear medium is given as $E^{(\omega)}=E_{1} \cos \left(k_{1} z-\omega t+\phi_{1}\right)$. When this wave propagates within the medium, it generates field at double the frequency given as $E^{(2 \omega)}=E_{2} \cos \left(k_{2} z-2 \omega t+\phi_{2}\right)$. From the above expressions, write the non-linear polarization terms at frequencies $\omega$ and $2 \omega$. If the velocity of non-linear polarization at $2 \omega$ is $\omega / k_{1}$ then write the velocity of EM wave at $2 \omega$ and arrive at the phase matching condition.
7. a) Derive laws of reflection from Fermat's principle.
b) Differentiate between uniaxial and biaxial crystals. Out of the seven basic crystal structures, which are classified as uniaxial and biaxial crystals?

## Part-B

Answer any 4 questions. Each carries 5 marks.
(5x4=20)
8. A convex lens and a concave lens, each of focal length ' $f$ ' are placed a distance ' $d$ ' ( $d<f$ ) apart with the convex lens placed to the left. Find the effective focal length of the system in terms of ' $f$ ' and ' $d$ '. If a parallel beam of light enters this lens combination system from the left, then at what distance from the second (concave) lens would it come to focus?
9. A converging lens system of focal length 40 cm , achromatic for the Fraunhofer C and F lines is to be constructed from two lenses- one of crown glass and other of flint glass. Find the effective focal length of these component lenses used to make the achromatic doublet. The refractive index for $\mathrm{C}, \mathrm{D}$ and F lines are given in the table below.

|  | C | D | F |
| :--- | :--- | :--- | :--- |
| Crown glass | 1.5145 | 1.5170 | 1.5230 |
| Flint glass | 1.6444 | 1.6520 | 1.6637 |

10. A circular scale of least count 1 second is fitted with the telescope objective of a spectrometer. The objective lens has a diameter of 1 inch. What is the resolution of the instrument if light of wavelength $6000 \AA$ is incident on it? With the given resolution of the circular scale, telescope objective of up to what diameter can be used? ( 1 inch=2.54 cm )
11. Two light beams having intensities in the ratio $1: 9$ produce interference fringes of visibility 0.3. What information do we get about the degree of coherence?
12. The function $\int_{-\infty}^{\infty}\left|f(t)^{2}\right| d t$ has infinite energy over the given integration range. Show that it still has finite average power. Now, for $f(t)=A \sin (\omega t+\epsilon)$, show that its autocorrelation function is given as $c_{f f}(\tau)=\left(A^{2} / 2\right) \cos \omega \tau$.
13. A phase retardation plate of quartz has a thickness of 0.1436 mm . Given that the refractive indices of quartz for ordinary and extra-ordinary wave are 1.5443 and 1.5533 respectively, calculate for which wavelengths of visible light (4000-7000 $\AA$ ) will it act like i) Quarter wave plate ii) Half wave plate.
