Register Number:
DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27

M.Sc. PHYSICS - I SEMESTER

SEMESTER EXAMINATION: OCTOBER 2021

(Examination conducted in January-March 2022)

PHBC 7120 – MATHEMATICAL PRELIMINARIES AND NEWTONIAN MECHANICS

(For Supplementary Candidates)

Time-1 1/2 hrs. Max Marks-35

This question paper has 2 printed pages

(**The question paper has two parts A& B. Answer any 3 questions from one part and 4 questions from the other part. Each question carries 5 Marks: 5x7=35**)

Part A

1.
   (a) Compute the $\vec{\nabla}\cdot\vec{F}$ and $\vec{\nabla}\times\vec{F}$ for $\vec{F}=x^2y\,\hat{\boldsymbol{i}}-\left(z^3-3x\right)\hat{\boldsymbol{j}}+4y^2\hat{\boldsymbol{k}}$.
   (b) If $\phi=3x^2y-y^3z^2$, find $\vec{\nabla}\phi$ at the point $(1,-2,-1)$. [3+2]
2. Consider the matrices: $A=\begin{pmatrix}7 & i\\ 0 & 1\end{pmatrix}$ and $B=\begin{pmatrix}1 & 0\\ 0 & 2i\end{pmatrix}$
   (a) Check whether A and B are Hermitian.
   (b) Find the eigenvalues and eigenvectors of A and B . [2+3]
3. Consider the following two kets: $|\psi\rangle=\begin{pmatrix}5i\\ 2\\ -i\end{pmatrix}$ and $|\phi\rangle=\begin{pmatrix}3\\ 8i\\ -9i\end{pmatrix}$
   (a) Find $|\psi\rangle^*$ and $\langle\psi|$.
   (b) Are $|\psi\rangle$ and $|\phi\rangle$ orthogonal?
   (c) Is $|\psi\rangle$ normalized? If not, normalize it. [2+2+1]
4. Consider the states $|\psi\rangle=2i|\phi_1\rangle+|\phi_2\rangle$ and $|\chi\rangle=-\frac{i}{\sqrt{2}}|\phi_1\rangle+\frac{1}{\sqrt{2}}|\phi_2\rangle$ where the two vectors $|\phi_1\rangle$ and $|\phi_2\rangle$ form an orthonormal basis:
   (a) Calculate the operators $|\psi\rangle\langle\chi|$ and $|\chi\rangle\langle\psi|$. Are they equal?
   (b) Find the Hermitian conjugates of $|\psi\rangle\langle\chi|$ and $|\chi\rangle\langle\psi|$. [3+2]
5. Find the values of C_1 and C_2 such that the function $f(z)=x^2+C_1y^2-2xy+i\left(C_2x^2-y^2+2xy\right)$ is analytic.

Part B

6.
(a) Define Instantaneous Velocity (in terms of vectors and calculus).
(b) A little girl is flying a drone using a remote controller. The drone's position is measured with respect to the little girl's position and varies as given by the equations:

$x = 2.0\,m - (0.25\,\mathrm{m/s^2})t^2$, $y = (1.0\,\mathrm{m/s})t + (0.025\,\mathrm{m/s^3})t^3$ and $z = 0.5\,\mathrm{m} - (4.9\,\mathrm{m/s^2})t^2$

i. Find a general expression for the drone's instantaneous velocity $\vec{\boldsymbol{v}}$.
ii. Express $\vec{\boldsymbol{v}}$ at $t = 2\,\mathrm{s}$ in component form and in terms of magnitude and direction.

[1+2+2]

7.
(a) An object undergoing projectile motion is thrown with a velocity of magnitude $|\vec{\boldsymbol{v}}|_{t=0} = v_0$ at an angle θ_0 with respect to the horizontal axis. Write down its equation of motion along the x and y directions.
(b) Obtain the expression for the horizontal range of the projectile.

8.
(a) State Newton's Second Law.
(b) An object is acted upon by two forces both of magnitude F . What would be the angle between the two forces if the magnitude of the net force is
i. $F^{\mathrm{NET}} = 1.8F$?
ii. $F^{\mathrm{NET}} = \sqrt{2}F$?

[1+2+2]

9. A high power vehicle (having its own thrust engine) is tested on an isolated, frictionless ground. To begin with, this vehicle of mass $50\,\mathrm{kg}$ it is at rest at the center of this ground and a force of $F = (20.3\,\mathrm{N/s})\ t$ (where t is time) is applied in the East direction (as you watch from the South to the North). How far does the object travel in the first $8\,\mathrm{s}$ after the force is applied?

10. A block of mass m is kept on the top of an inclined plane of height h and length ℓ (along the inclined plane). What is the time taken by the block to slide from the top of the plane from rest to its bottom (obtain the expression in terms of h and ℓ)?